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# Model for a Nonlinear Tank System Under Proportional-Integral-Derivative Control

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# MODEL FOR A NONLINEAR TANK SYSTEM UNDER PROPORTIONAL-INTEGRAL-DERIVATIVE CONTROL

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ABSTRACT

A model (NONLINRK) was developed for a closed tank system under feedback control by an ideal proportional-integral-derivative controller. Under servo action the fluid level in the tank is altered from its equilibrium setpoint. Under regulator action the feed pressure to the inlet valve and/or the outlet valve percentage opening are varied from equilibrium settings. The numerical model uses Gill's fourth-order Runge-Kutta algorithm to solve the system equation. The equation was made separable by approximating an exponential factor by the tangent at the beginning of each time step in the numerical solution.

NONLINRK simulation trials exhibited many characteristics of nonlinear systems including unequal offset under proportional control for setpoint changes equal in magnitude but opposite in sign, harmonics in the response to a sine wave input on fluid level setpoint and bounded response in spite of increased gain settings. In addition, further simulation trials showed the system response converges to that of a linear system for sufficiently small setpoint or load variations.

A second model using the modeling language TUTSIM provided corroboration of the results produced by NONLINRK.

Proportional and proportional-integral control simulations differed by less than .1% and the models showed the same rates of convergence as the time step was decreased. Under PID control TUTSIM simulations developed severe instabilities, but NONLINRK exhibited the expected trends in the increased ability to react to a ramp function disturbance and the decrease in phase lag in response to a sinusoidal setpoint function.

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# INTRODUCTION

Process control systems may be classified in two groups, linear or nonlinear. Linear control systems may be recognized by the form of the system of differential equations which is employed to model their dynamic behavior. Each term in the system of equations is of the form of a constant multiplied by the dependent variable or its derivatives. Equations of this form can always be solved analytically. Linear models may provide reasonably accurate results for wide ranges of operation for some systems, but all real systems are inherently nonlinear (Graham and McRuer 1971). For example, a spring and damper system may be modeled as linear, assuming an ideal Hooke's law behavior for the spring, then the system equation contains the linear term spring constant times the displacement (the dependent variable). If the spring is sufficiently stretched or compressed then the spring behavior is a function of displacement and the term describing spring action is expressed as the product of the dependent variable and a function of the dependent variable, a nonlinear form.

Nonlinear control system models may contain functions of the dependent variable, products of functions of the dependent variable and the independent variable, saturation or limiting restrictions, preload, threshold, rectifier and on-off nonlinearities to name some of the typical forms. The tank system modeled in this work is represented by equations which include the first three examples of nonlinearities listed above. The difficulty with nonlinear system models is that analytical solutions for nonlinear differential equations are quite rare. For this reason, a numerical approach is chosen to model the system. Such solution techniques may be versatile but they do not provide a means for gaining a deeper insight to general properties of the system and its modes of response.

# CHAPTER 1 THE TANK SYSTEM

The system consists of a tank, an inlet valve under feedback control and a manually operated discharge valve. The inlet valve controller is modeled as an ideal proportional integral-derivative (PID) unit with the difference between the desired tank pressure and the actual tank pressure as the error signal. Figure 1 shows the components and layout of the system.

The system features are explained in the following sequence which traces flow from inlet to outlet. The fluid flow rate at the inlet is proportional to the inlet valve opening and a function of the difference between the inlet pressure and the pressure in the tank. The inlet valve is driven by a signal from an ideal PID controller, whose inputs are the tank pressure and the desired pressure. The controller computes an error signal (the desired pressure minus the tank pressure), the integral of the error, and the derivative of the error. The controller sends a signal to the valve to change its opening based upon the values of these three quantities. The system operator may set the controller to determine how strongly each of the three actions - proportional, integral or derivative, is to influence the valve setting. The controller parameters are defined as the proportional gain ($K_c$), reset rate ($R_r$) and

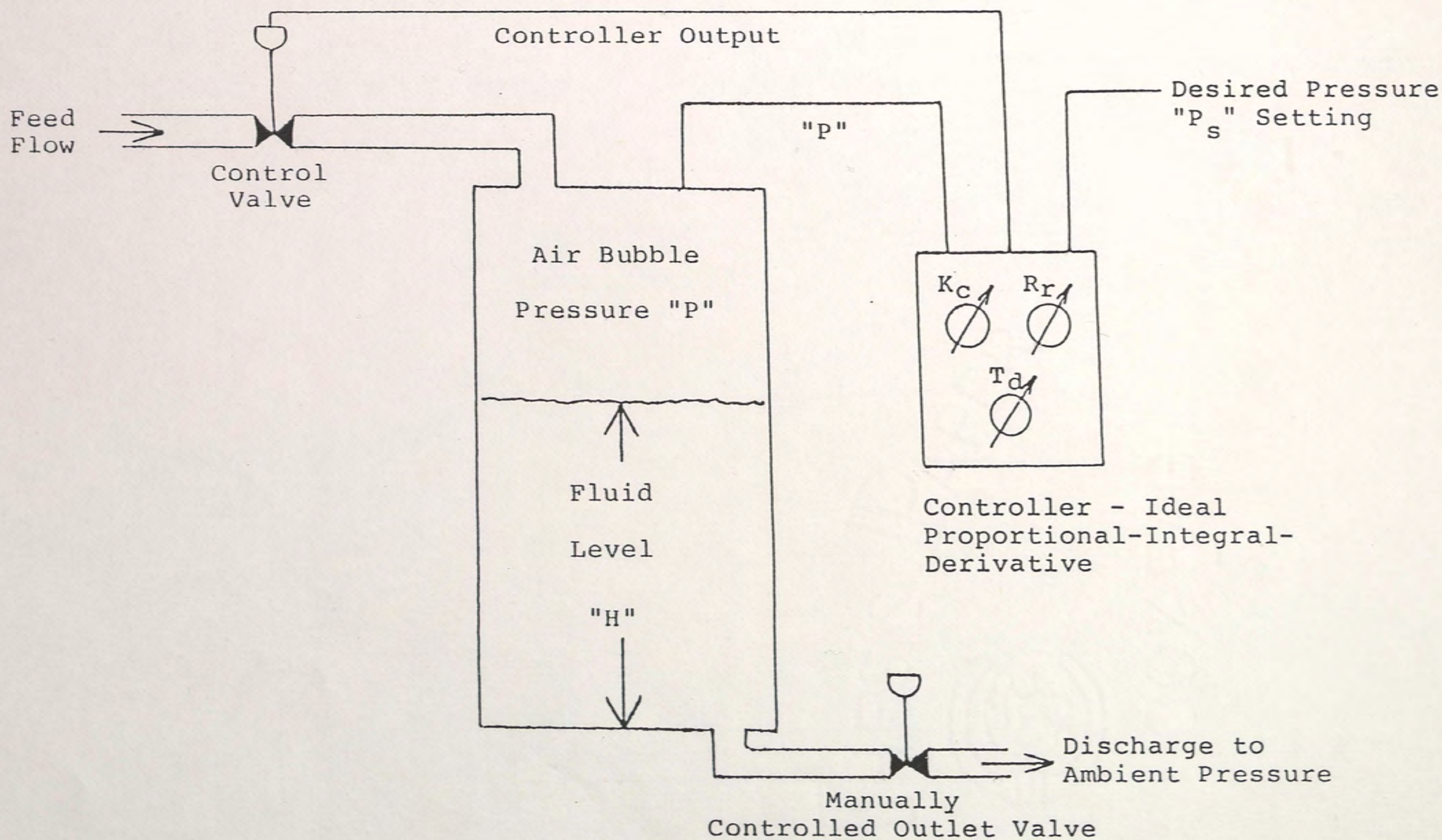


Figure 1. Components and Layout of the Tank System.

derivative time ($T_d$) respectively. In general the influence of each action on the inlet valve opening increases as the corresponding parameter is increased.

The flow passes next from the inlet valve into the tank. It is presumed that the tank initially contained air at 14.7 psi absolute and that as fluid fills the tank the pressure in the air above the tank increases as the air volume decreases according to Boyle's Law. The tank is drained by a manually controlled outlet valve. The flow is proportional to the valve opening and a function of the tank pressure plus the hydraulic head within the tank.

Figure 2 shows a control system, the Technovate model 9030, which incorporates all the features outlined above, save the ideal PID controller. The controller action is, however, quite similar to the ideal case (Shinskey 1967) where $T_d$ and/or $R_r$ are zero (proportional (P) control, proportional-integral (PI), proportional-derivative control).

The controller output is pneumatic with pressure ranging between zero and 20 psi supply. The Technovate inlet valve is an air-to-open, equal percentage valve which responds to controller pressures between 3 and 15 psi. At 3 psi the valve is closed and 15 psi input causes the valve to open fully. The outlet valve opening is calibrated from 0 to 100 percent and may be manually controlled.

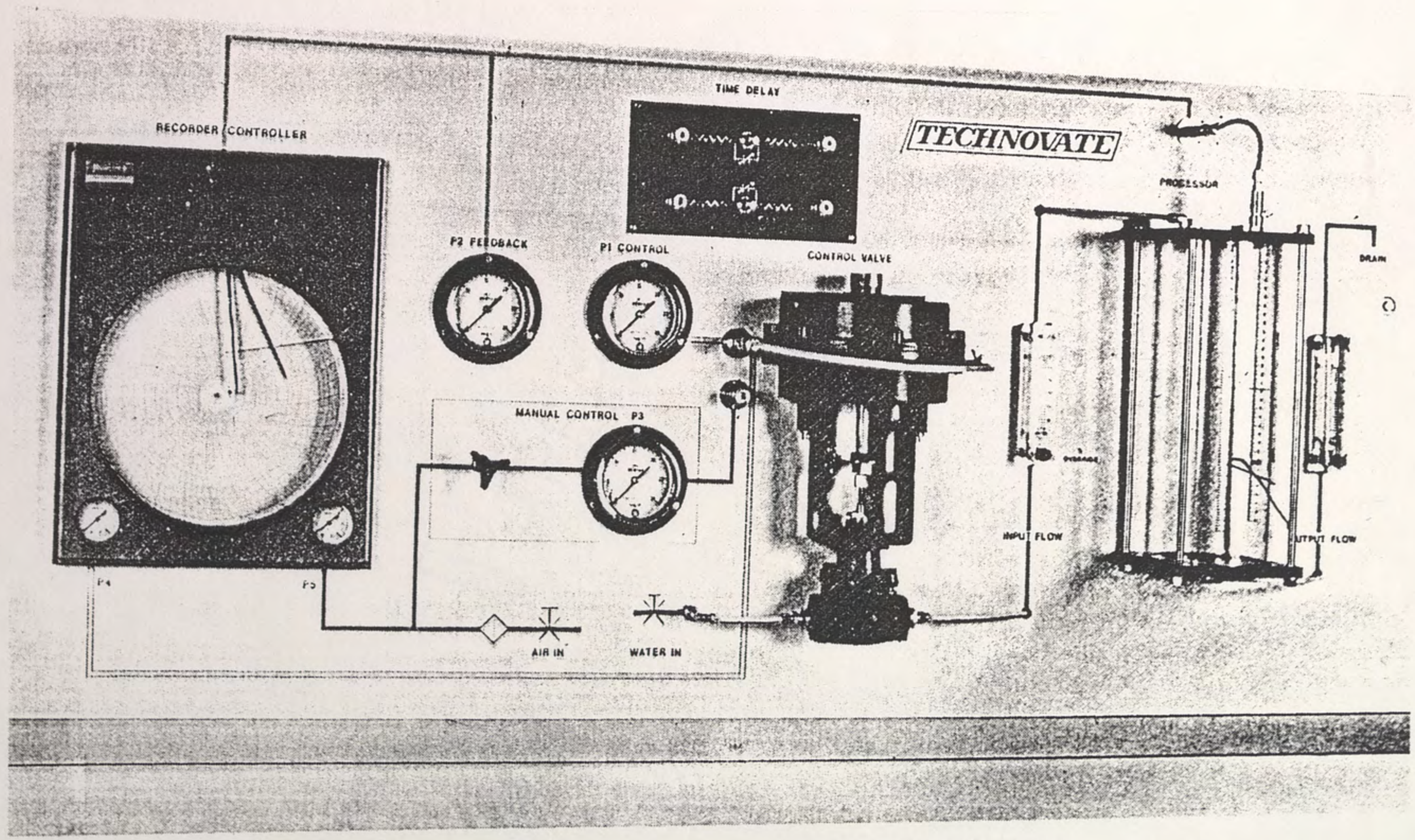


Figure 2. Technovate Model 9030 Laboratory Scale Control System.

The nonlinear system model that is developed in the next chapter corresponds closely to the Technovate system. The only significant difference is in the controller. The system model is versatile in that it is formulated to allow the modeler to select various tank sizes, equal percentage inlet valve ratings and outlet valve sizes.

The modeled system controlled variable is the pressure in the tank. The setpoint is then the desired pressure in the tank and when this pressure ($P_s$) is changed the system is operating under servo control. The setpoint may also be specified in terms of the fluid level in the tank. Given a specific tank height, a simple units conversion formula may be used by the operator to convert the desired fluid level to a pressure setting.

Under regulator action the system's two load variables are the pressure on the feed side of the inlet valve and the outlet valve setting in the percentage opened.

In practice, systems such as this find wide application in flow control. The tank provides a capacitive-type element which is slow to respond to variations in input feed pressure. This allows for a well-regulated output flow. Another application could be in a chemical batch process unit, where the tank is actually a reaction vessel operating under specific pressure requirements, and the outlet flow may be the product or may be a combination of product and waste streams.

# CHAPTER 2 MATHEMATICAL DESCRIPTION OF THE TANK SYSTEM

The 5 key elements in the system are:

1) the inlet valve reaction to control pressure,
2) the fluid flow rate into the tank,
3) the pressure in the tank or the fluid level,
4) the flow rate out of the tank and
5) the controller action.

The equations to model these elements are developed in the above order.

The equal percentage inlet valve is modeled according to the type of action that this valve is expected to deliver. Such valves are characterized by their equal percentage rating and a valve size parameter. The valve reaction to change in control pressure is computed by multiplying the equal percent rating by the percentage change in the control pressure to obtain the percentage change in valve opening (Weber 1973). For example an equal percentage valve with a "5 percent" rating would deliver a 5 percent change in the valve opening when the control pressure changes by 1 percent. The flow coefficient, $C_v$ or valve size parameter, is specified in units of lbs/minute of water that flow through a fully open valve at 60°F when the pressure drop is 1 psi. Fluids with similar viscosity may also be modeled using $C_v$

corrected for fluid density and the proper fluid density must be used throughout the model. The equal percentage valve fractional opening, $A_v$, then is characterized by the following equation:

$$\frac{dA_v}{A_v} = K \frac{dP_v}{12} \qquad [1]$$

where K is the equal percent rating and $P_v$ is the control pressure.

This equation shows that the fractional change in $A_v$ is equal to a constant times the fractional change in $P_v$, where the 12 is the maximum allowable change in $P_v$. From the system description we find when $P_v$ is 15, $A_v$ is 1 (full open), then Equation 1 may be integrated from $A_v$ equals 1 to $A_v$' and $P_v$ equals 15 to $P_v$' to obtain $A_v$ as a function of control pressure, $P_v$.

$$\int_1^{A_v'} \frac{dA_v}{A_v} = \int_{15}^{P_v'} K \frac{dP_v}{12} \qquad [2]$$

This is recognized as the familiar exponential function

$$A_v = e^{K/12(P_v - 15)} \qquad [3]$$

Where the primes have been dropped since there is no need to distinguish the dummy variables of integration as in Equation 2. The complete the functional form of the inlet valve action the flow coefficient $C_v$ is multiplied by $A_v$, giving the inlet valve function $C_i(P_v)$ in units of lb/min flow:

$$C_i(P_v) = C_v e^{K/12(P_v - 15)} \qquad [4]$$

The next element to model is the flow rate into the tank. Bernoulli's equation, for the case when the flow velocity downstream from the valve is much greater than upstream, indicates that the flow is proportional to the square root of the pressure drop across the valve. The parameter $C_v$ has been specified in units of flow rate (lbs/min), so the units of the Bernoulli term are neglected and its product with Equation 4 defines the flow in $F_i$:

$$F_i = C_v e^{K/12(P_v - 15)} \sqrt{P_i - P} \qquad [5]$$

where $P_i$ is the fluid supply pressure and the units of $F_i$ then are lbs/minute.

The pressure in the tank is modeled using Boyle's Law. For a cylindrical or rectangular tank the pressure of the air in the tank is given by:

$$P'V' = PV \qquad [6]$$

The primed symbols refer to another state of the same quantity of air. With an empty tank having height, $H_t$, the pressure is 14.7 psi absolute. When the tank is filled to a level H, the pressure is given by:

$$P = \frac{14.7\ H_t}{H_t - H} \qquad [7]$$

Using gauge pressure throughout this work Equation 7 becomes upon subtraction of 14.7 psi:

$$P = \frac{14.7\ H}{H_t - H} \qquad [8]$$

The outlet flow is proportional to the square root of the pressure difference from the bottom of the tank to ambient air. The pressure at the bottom of the tank is the sum of the pressure in the air above the tank, P, and the pressure due to the fluid level above the tank bottom. The ambient air pressure is zero since gauge pressures are employed in the model.

The resultant Bernoulli factor is $\sqrt{\gamma H + P}$ where $\gamma$ = fluid density in lb/in$^3$. As with the inlet valve, a flow coefficient, defined as $C_\theta$, is the flow rate in lbs/min at 60°F, 1 psi pressure drop for the fully opened valve. The flow at openings less than full is the percentage open, $\theta$, divided by 100, then multiplied by $C_\theta$. In general then the outlet flow $F_o$, is given by:

$$F_o = C_\theta \ \frac{\theta}{100} \sqrt{\gamma H + P} \qquad [9]$$

The last element to consider is the controller. For an ideal PID controller the output pressure, $P_v$, is given by:

$$P_v = \bar{P}_v + \Delta P_v; \ 3 \leq P_v \leq 15; \text{ and} \qquad [10]$$

$$\Delta P_v = K_c \left[ (P_s - P) + R_r \int_0^t (P_s - P)\, dt + T_d \frac{d(P_s - P)}{dt} \right] \qquad [10a]$$

$\overline{P}_v$ is a value for the controller pressure at an initial steady state condition. The lower limit restriction on $P_v$, setting a minimum of 3 psi, leads to a problem in the model of the valve opening function in Equation 3. When $P_v$ is 3 psi, $A_v$ is $e^{-K}$, not 0. However, for sufficiently large K values, $A_v$ is very small. In this work a base case with K equal to 5 yields an $A_v$ of .0067 which closely approximates a closed valve. If a system with a smaller value of K were modeled, then it may be better to depart from the exponential function when $P_v$ is near 3 psi and then follow a linear function which gives an $A_v$ of 0 when $P_v$ is 3 psi.

This completes the set of equations which are employed to describe each system component. The equation which describes the dynamics of the controlled variable, P, is derived from a mass balance on the flows in and out of the tank. That is, the rate at which fluid accumulates in the tank is equal to the inlet flow minus the outlet flow.

$$\gamma A \frac{dH}{dt} = F_i - F_o \qquad [11]$$

Where A is the tank cross-sectional area and $\gamma$ the fluid density.

Equation 11 describes the rate at which H, the tank fluid level, changes in response to inlet and outlet flows. This equation is expressed in terms of the pressure using the substitution from Equation 8:

$$H = PH_t/(14.7 + P) \qquad [12]$$

Differentiating both sides with respect to time:

$$\frac{dH}{dt} = \frac{14.7H_t}{(14.7+P)^2} \frac{dP}{dt} \qquad [13]$$

Combining equations 5, 9, 12 and 13:

$$\frac{dP}{dt} = \frac{(14.7+P)^2}{14.7H_t\rho A} \left[ C_v e^{K/12(P_v-15)} \sqrt{P_i - P} \right. \qquad [14]$$

$$\left. - C_\theta \frac{\theta}{100} \sqrt{\frac{\gamma H_t P}{14.7+P} + P} \right]$$

Equations 10, 10a and 14 represent the dynamic behavior of this system as it is modeled in subsequent chapters.

These equations contain several nonlinearities. The controller function $P_v$ has limiting nonlinearities at 3 psi and 15 psi. $P_v$ is otherwise a linear function of the dependent variable P, it's derivative, the integral of P and the independent variable t. The variable, $P_s$, its derivative and integral are merely time dependent functions. Equation 14 however, has $P_v$, hence the dependent variable, its derivative and integral, in an exponential function.

This presents a nonlinearity that is difficult to deal with even in a numerical solution technique. The differential equation, 14, for P is implicit in its highest-order derivative and cannot be solved for explicitly, as required in Runge-Kutta, predictor-corrector and Newton-Cotes integration techniques (Burden, Faires and Reynolds 1980).

The remaining nonlinear forms in equation 14 involve terms where the square of P, the square root of P and the inverse of P appear. Thus several nonlinear functions of the dependent variable are evident in Equation 14.

This model omits some system features which may be of interest. Delay in the control pressure line (due principally to the line capacitance) and valve dynamics, e.g., friction, inertia and spring (oscillatory) effects have not been modeled. This limits the applicability of the model somewhat, but a control system should be designed to minimize these factors under the normal modes of operation. The model also neglects the effect of head losses due to flow resistance in the valves and piping.

## CHAPTER 3 THE NUMERICAL MODEL

Equation 14 is restated below in modified form to facilitate the discussion of the numerical model.

$$\dot{P} = f_1 e^{f2} - f_3 \qquad [15]$$

Where functions $f_1$, $f_2$ and $f_3$ are defined as:

$$f_1(P,t) = \frac{(14.7+P)^2}{14.7 H_t \rho A} \; C_v \sqrt{P_i\,(t) - P} \qquad [15a]$$

$$f_2(\dot{P},P,\int P,t) = \frac{K}{12}\,[\bar{P}_v - 15 + K_c(P_s(t) - P + R_r\int_o^t P_s(t')dt' - R_r\int_o^t P dt' + T_d\dot{P}_s(t) - T_d\dot{P})\,] \qquad [15b]$$

$$f_3(P,t) = C_\theta\,\frac{\theta(t)}{100}\sqrt{\frac{\gamma H_t P}{14.7+P} + P} \quad \frac{(14.7+P)^2}{14.7 H_t \rho A} \qquad [15c]$$

The dots in the above expressions signify differentiation and a lower case t in parenthesis signifies that the preceding symbol pertains to a known function of time. All other symbols are as previously defined.

Several approaches were considered for solving this system. The usual linearization about one operating point or known system state was rejected since simple hand calculations show that a linearization of the inlet valve action would be greatly in error at the extremes of the valve opening. As stated previously, numerical methods are not immediately applicable due to the implicit form of Equation 15. To resolve this difficulty it was decided to linearize the factor, $e^{f_2}$, and to relinearize as the solution proceeds along the exponential, effectively breaking it into many straight segments. Then if $f_2$ does not change greatly from one step to the next the linear approximation may be reasonably accurate. The linearization step is then:

$$e^{f_2} \approx A + Bf_2 \qquad [16]$$

Substituting this approximation into Equation 15 allows $\dot{P}$ to be separated. The integral of P in $f_2$ remains to be considered. Two alternatives for treating this problem were evident. The system equation may be differentiated, resulting in a very complex second order equation with the extra requirement that $\ddot{\theta}$ and $\dot{P}_i$ be evaluated. The second alternative considered was to evaluate the integral numerically. This approach was found to work reasonably well since the integral of the error changes slowly for the system. In feedback control the integral term is a measure of accumulated errors and should not be subject to rapid variations. This second alternative can be implemented much more easily than the first and if a simple trapezoidal integration is used, then the algorithm is self-starting. More sophisticated integrating techniques, for example Simpson's rule or Adams-Bashforth, could be used if the assumption that the integral of the error is slowly varying is not adequate. The trapezoidal rule for the integral of P is:

$$\int_{o}^{t_n} P dt \approx \left[ \frac{P_o}{2} + \sum_{k=1}^{n-1} P_k + \frac{P_n}{2} \right] \Delta t \qquad [17]$$

Using equations 16 and 17 in Equation 15 and separating $\dot{P}$ we have the following approximation for $\dot{P}$:

$$\dot{P} = f_4 f_1 \ (A + Bf_5) - f_4 f_3 = F(p,t) \qquad [18]$$

Where $f_4$ and $f_5$ are defined as:

$$f_4(P,t) = \left(1 + \frac{f_1 BK \ K_c T_d}{12}\right)^{-1} \qquad [18a]$$

$$f_5(P,t) = \frac{K}{12} \ [\overline{P}_v - 15 + K_c(P_s(t) - P + R_r \int_0^{t_n} P_s(t)dt$$

$$-R_r(\frac{P_o}{2} + \sum_{k=1}^{n-1} P_k + \frac{P_n}{2}) \ \Delta t + t_d \dot{P}_s(t))] \qquad [18b]$$

This form of the system is amenable to many numerical integration techniques. Gill's fourth order Runge-Kutta was chosen for the model (Carnahan, Luther, and Wilkes 1969), and is shown in equations 19 below.

$$P_{n+1} = P_n + \frac{\Delta t}{6}(K_1 + (2-\sqrt{2})K_2 + (2+\sqrt{2})K_3 + K_4) \qquad [19]$$

Where $K_1$, $K_2$, $K_3$ and $K_4$ are defined as:

$$K_1 = F(P_n, t_n) \qquad [19a]$$

$$K_2 = F(P_n + \frac{\Delta t}{2}K_1, t_n + \frac{\Delta t}{2}) \qquad [19b]$$

$$K_3 = F(P_n + (\frac{1}{\sqrt{2}} - \frac{1}{2})K_1\Delta t + (1-\frac{1}{\sqrt{2}})K_2\Delta t, t_n + \frac{\Delta t}{2}) \qquad [19c]$$

$$K_4 = F(P_n - \frac{\Delta t}{\sqrt{2}}K_2 + (1+\frac{1}{\sqrt{2}})K_3, t_n + \Delta t) \qquad [19d]$$

There was no need to resort to a more economical method in terms of computer time since the Prime 400 unit used in this work was reasonably fast. If this model was executed on a microcomputer, then it would be advisable to use an explicit Adams-Bashforth or predictor-corrector method to decrease the run time.

The solution of Equation 18 is started by fitting a straight line tangent to the exponential function in Equation 15 for a known initial value of $f_2$. At the beginning of the time step $f_2$ is computed, then the line tangent to $e^{f_2}$ goes through the point $(f_2, e^{f_2})$. As an aside it must be noted that $f_2$ is a bounded function since by equations 4 and 15 we have:

$$e^{f_2} = e^{K/12(P_v - 15)}; \; 3 \le P_v \le 15 \qquad [20]$$

Then the limits of $f_2$ are $-K \leq f_2 \leq 0$ and K, the equal percentage rating, is typically less than 10 for most val es. Returning to the process of approximating Equation 20 by its tangent, the most recently computed value for $f_2$ is used to find the equation of the tangent line by the point-slope method. Therefore the coefficients A and B in Equation 16 are determined by evaluating:

$$A = \left[ e^{f_2} \Big|_{t_i} \right] \left( 1 - f_2 \Big|_{t_i} \right) \quad \text{and} \qquad [21]$$

$$B = \frac{d}{df_2} \left(e^{f_2}\right) \Big|_{t_i} = e^{f_2} \Big|_{t_i} \qquad [22]$$

The vertical line above the symbol $t_i$ represents the value of $f_2$ at the beginning of the time step. Figure 3 shows the exponential function for a valve with an equal percentage rating of 5. When $f_2$ is -2 (corresponding to a $P_v$ of 10.2 psi) the straight line approximation is:

$$\begin{aligned} e^{f_2} &\approx e^{-2}(1 - (-2)) + e^{-2} f_2 \\ &\approx .406 + .1353\ f_2 \end{aligned}$$

Using the above example it is found that if $P_v$ changes by .1 psi over the next time step, the straight line approximation and the value of the exponential differ by

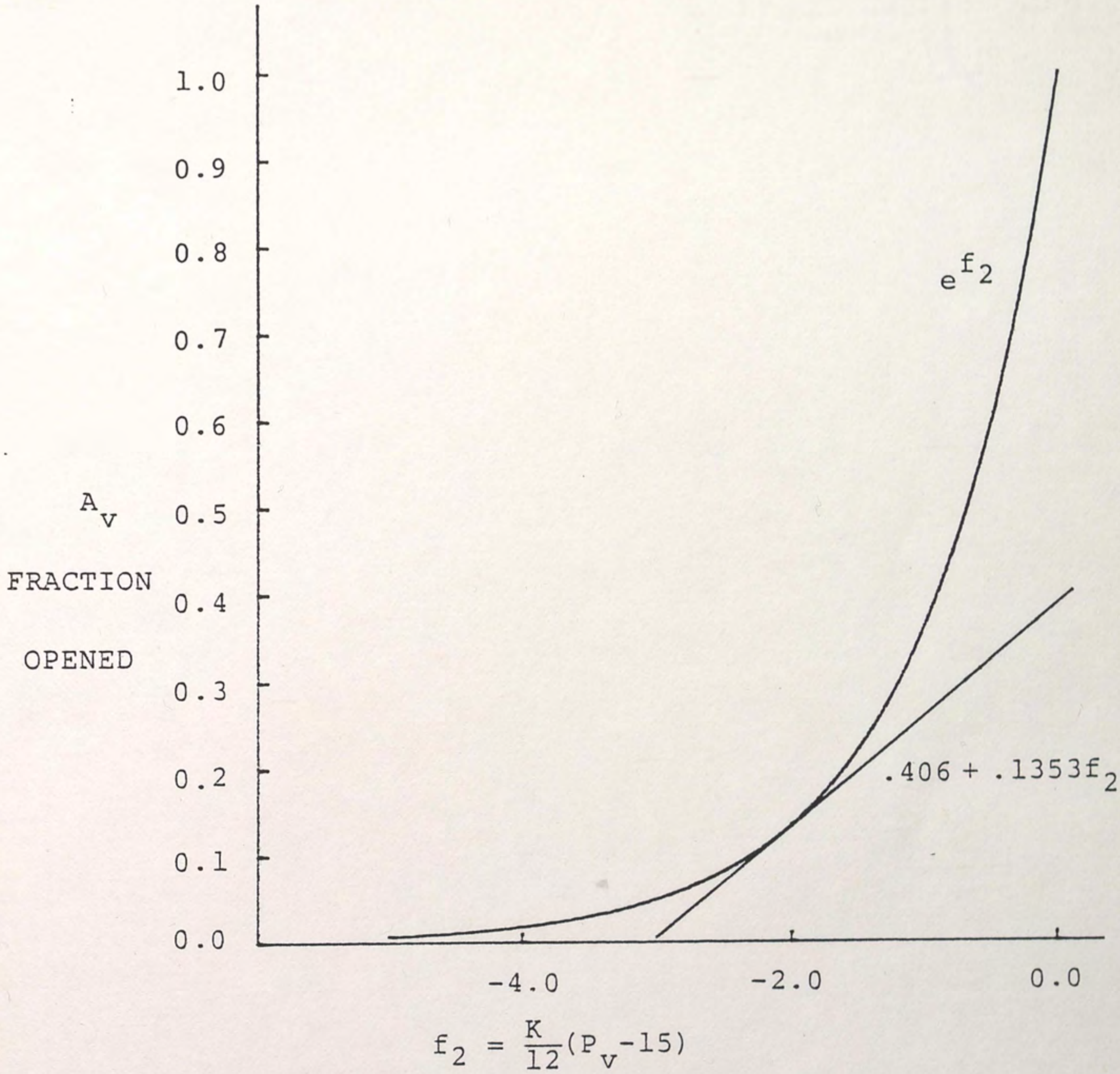


Figure 3. The Valve Opening Function and a Linear Approximation for the Case of $f_2$ Equal to -2 at the Beginning of a Time Step.

less than .1%. A change in $P_v$ of 1 psi from the base value would show a 5% difference between the approximation and the exponential function. Thus for small changes in $P_v$ the linear function is a good approximation of the exponential. Since a new linear function is found at each time step, a reduction in time step should provide a more accurate result if, in a particular simulation, large changes in $P_v$ cause significant errors due to the linear approximation.

The system may now be solved with the Gill's algorithm, the result being the value of P at the next time step. The new value of P is used to compute $f_2$. Then a new line is fit to the exponential function and the model proceeds in this loop until the end of the simulation time is reached.

The model requires many inputs:

1) Supply pressure in psi,
2) Control valve flow coefficient in lbs/min,
3) Control valve equal percentage rating,
4) Steady state control pressure, in psi, just prior to regulator and/or servo action,
5) Initial fluid level in inches,
6) Initial outlet percentage opened,
7) Controller proportional gain, reset rate and derivative time,
8) Time step in seconds, the number of time steps to the end, and the number of time steps per printed result,

9) Time dependent functions describing the servo and/or regulator actions (setpoint, inlet pressure and outlet opening) and

10) Tank height and radius.

The model is based upon steady state initial conditions. The outlet valve flow coefficient is computed in the model using the given input.

The input data listed in items 1 through 8 above may be entered interactively or in data statements. The servo and regulator functions in item 9 must be coded into the function subroutines:

1) SET(t), setpoint pressure in psi,

2) SETDER(t), the derivative of item 1 above,

3) SETINT(t), the integral of item 1 above,

4) PSIN(t), the inlet pressure in psi and

5) THETA(t), the outlet opening in percentage.

The time units are in minutes for these functions. The function SET(t) is restricted to continuous functions when $T_d$ is not zero. This restricts the use of the model to proportional-integral (PI) control for servo simulations with step or pulse type changes in setpoint. This restriction should not present any problem in practice, since it is widely accepted that derivative control works poorly for step changes in setpoint (Shinskey 1967).

The output of the model is in tabular form showing the time since the start of the simulation, the fluid level, tank pressure, control valve pressure, the derivative of pressure, $R_r$ times the integral of the error, the linear fit parameters A and B, the inlet flow rate and outlet flow rate. These outputs are listed as ten columns across the page. Appendix I contains a sample listing of the model output.

# CHAPTER 4 SIMULATION RESULTS

Among the large variety of potential input functions which may be used to explore the model and its predictions of system behavior, a few were selected. Sine functions with various amplitudes and period are used in servo and regulator control cases, to explore the frequency response of the system model. Step functions, where appropriate, are also used in servo and regulator control to simulate sudden decisions to change the pressure or outlet flow, both of which are under manual control. Another function used is the classic test of system response to a pulse type disturbance, which is expected to cause a temporary disturbance in the controlled variable and eventual return to equilibrium. Ramp functions on the setpoint and outlet opening are also modeled. The ability of the system to follow the ramp and minimization of overshoot when under integral control (compared to a step change) is demonstrated.

Considering the number of variables involved, a limited parametric study of the system model was undertaken. A base case was formulated, as shown in Table 1, and the system model was tested for the functions described above, and for variations in proportional gain, reset rate, derivative time and initial fluid level. Table II shows the

TABLE I

BASE CASE SYSTEM PARAMETERS

| PARAMETER | VALUE |
| --- | --- |
| Tank Height | 120 inches |
| Tank Radius | 18 inches |
| Inlet Valve Flow Coefficient | 50 lb/min |
| Inlet Valve Equal Percent Rating | 5% |
| Fluid Density | .03614 $lb/in^3$ |
| $\overline{P}_v$ Initial Control Pressure | 9 psi |

TABLE II

BOUNDED PARAMETERS

| PARAMETER | LOWER LIMIT | UPPER LIMIT |
| --- | --- | --- |
| Inlet psi ($P_i$) | 75 | 125 psi |
| Initial Fluid Level in Inches | 0.0 | $\frac{120P_i}{(14.7+P_i)}$ |
| Outlet Open | 0.0 | 100% |
| Tank Pressure | 0.0 | $P_i$ |
| Control Pressure | 3.0 | 15 psi |

limits placed on some of the parameters as a result of the base case assumptions and system design.

Before presenting the specific simulation results, some comments on the response of nonlinear systems in general and the expected response of this particular closed tank system are in order. When a linear system is forced with a sine wave the expected response (after transients have died down) is a sine wave of identical frequency but the amplitude and phase may be altered. A nonlinear system, however, may respond with new frequencies which are harmonics or subharmonics of the forcing function (Graham and McRuer 1971). In addition, a nonlinear system with such frequency response should approach the linear case as the amplitude of the forcing function approaches zero. This is owing to the fact that for sufficiently small disturbances a first order or linear approximation to a nonlinear system is quite accurate.

Another system characteristic is offset, a steady state error that is approached following a step change input. In linear systems under proportional control and subjected to a step type forcing function the magnitude of the offset is equivalent for positive or negative values of the same step increment, and the steady state is approached at the same rate whether the step is negative or positive. Nonlinear systems, on the other hand, may respond with unequal offsets and at different rates for the case of a given step up versus the same step down. As in the preceding sine function discussion, for a sufficiently small step the nonlinear system approaches the linear. These general system response characteristics are explored in the model simulation.

The nature of the tank system response in terms of the relative sensitivity to the three forcing functions (setpoint, outlet percentage opened and inlet feed pressure) may also be determined. Referring to Equation 11,

$$\gamma A \ \frac{dH}{dt} = F_i - F_o, \tag{11}$$

we may consider the potential impact of the three functions upon the terms $F_i$ and $F_o$ in light of system limitations and the base case parameters stated above. At initial steady state conditions the derivative in Equation 11 is zero, then the relative effect of a step change in each of the three forcing functions upon dH/dt is a measure of system sensitivity. Consider first an inlet feed pressure step change, since it is the simplest to analyze. Inlet feed in the base case is expected to be 100 psi. If it is assumed that the supply is regulated to within 25 percent of this value and the tank is normally about half full then a step change of 25 psi would initially alter the term $F_i$ by roughly 20%, with no initial change in $F_o$.

Next consider a step change in outlet percentage opened. This parameter in its extreme of variation may initially be at a very low setting when a call for full production is issued. Thus, if 10% is chosen as a reasonable low setting and the valve is opened fully then the term $F_o$ would initially undergo a ten-fold increase.

The setpoint function appears in the exponential factor of $F_i$, and if we consider an initial state with a near empty tank and steady state control pressure near 3 psi, then an increase in setpoint could easily drive the valve to saturation increasing $F_i$ by roughly a factor of 100 (depending upon the proportional gain selected).

Using these rough estimates as a measure of expected sensitivity to a range of reasonable inputs, the order of sensitivity of the controlled variable to the input functions is: setpoint (highest), outlet percentage and inlet feed pressure.

The crude analyses above provide a framework in which to evaluate the system model. Being a nonlinear system, direct verification of model results is difficult. Chapter 5 explores the verification of the model by comparison to a model developed using a software package designed specifically for continuous dynamic systems simulation.

In addition to a framework for evaluation, the above analysis identify setpoint and outlet percentage opened as the functions having major impact on the system. Therefore forcing functions of these two inputs comprise the majority of the simulations that follow.

The two curves on Figure 4 show the nonlinear system response for a servo, proportional-integral control simulation. Sine functions with period 5 minutes and amplitudes of 1.153 psi and .220 psi (corresponding to level increases of 5 inches and 1 inch respectively above a base of 30 inches) force the oscillations shown in the figure. The system response is clearly nonlinear for the higher amplitude sine function. The lower amplitude forcing function response resembles a sine wave, however, closer examination reveals that the half-period times are 2.3 and 2.7 minutes long, not 2.5 minutes as would be found for the steady state response of a linear system to this sine wave input. In Figure 5 the sine wave amplitude is reduced to .0456 psi and this discrepancy is eliminated. After the transients pass, the system settles down to an oscillating mode with a very symmetrical shape, much closer to a sine wave than the responses shown in Figure 4. In addition, Figure 5 includes a curve showing the model response when derivative control is added. As expected, the model predicts that derivative control decreases the phase shift, and in this case, with $T_d$ equal to one, the phase is decreased by approximately .8 minutes or $60^{\circ}$.

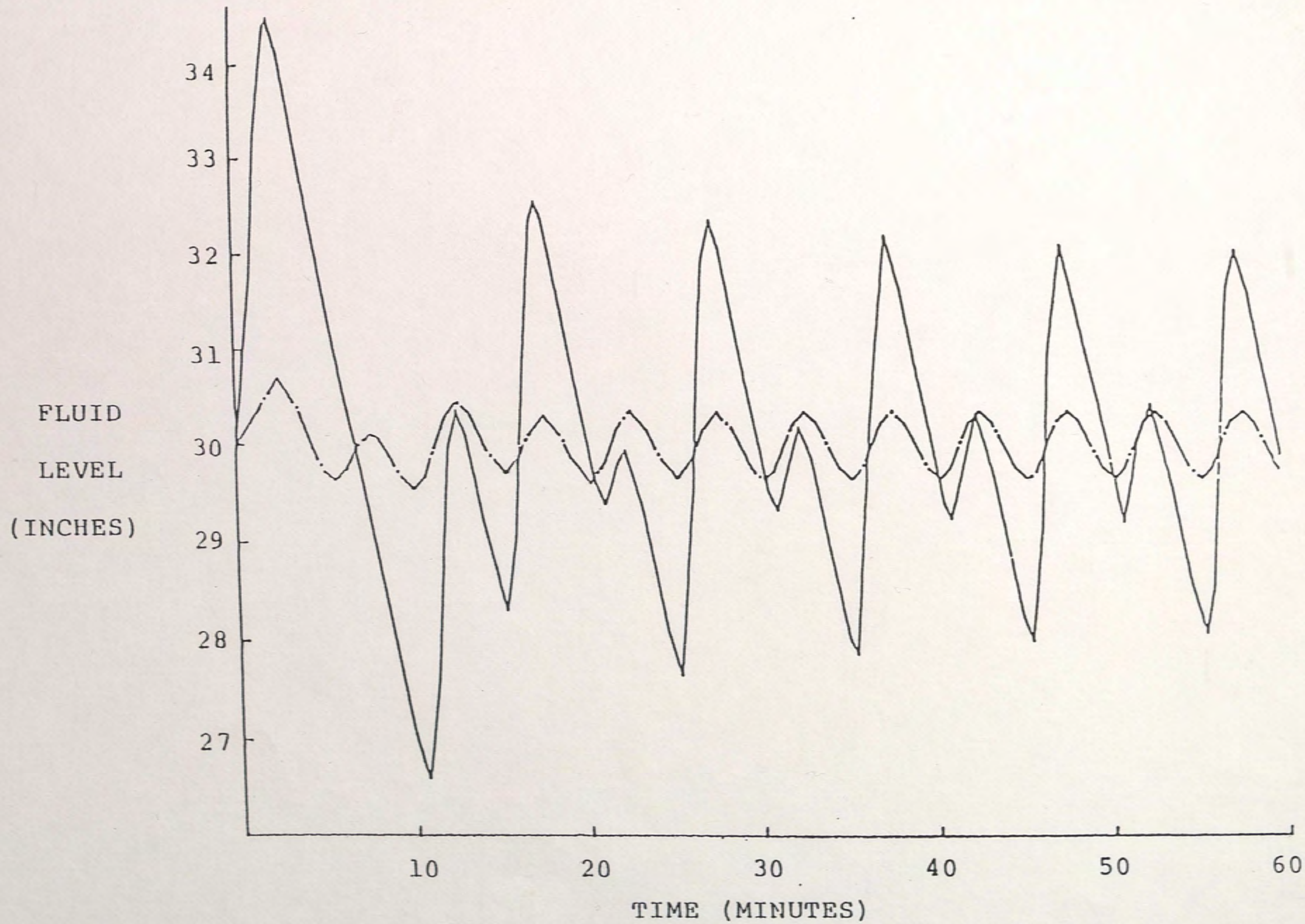


Figure 4. Level Responses to Sine Input on the Setpoint Pressure. Solid Line is the Response to a 1.153-psi Amplitude and Dash-Dot Line is the Response to a .220-psi Amplitude Sine Wave ($K_c$=3, $R_r$=.5).

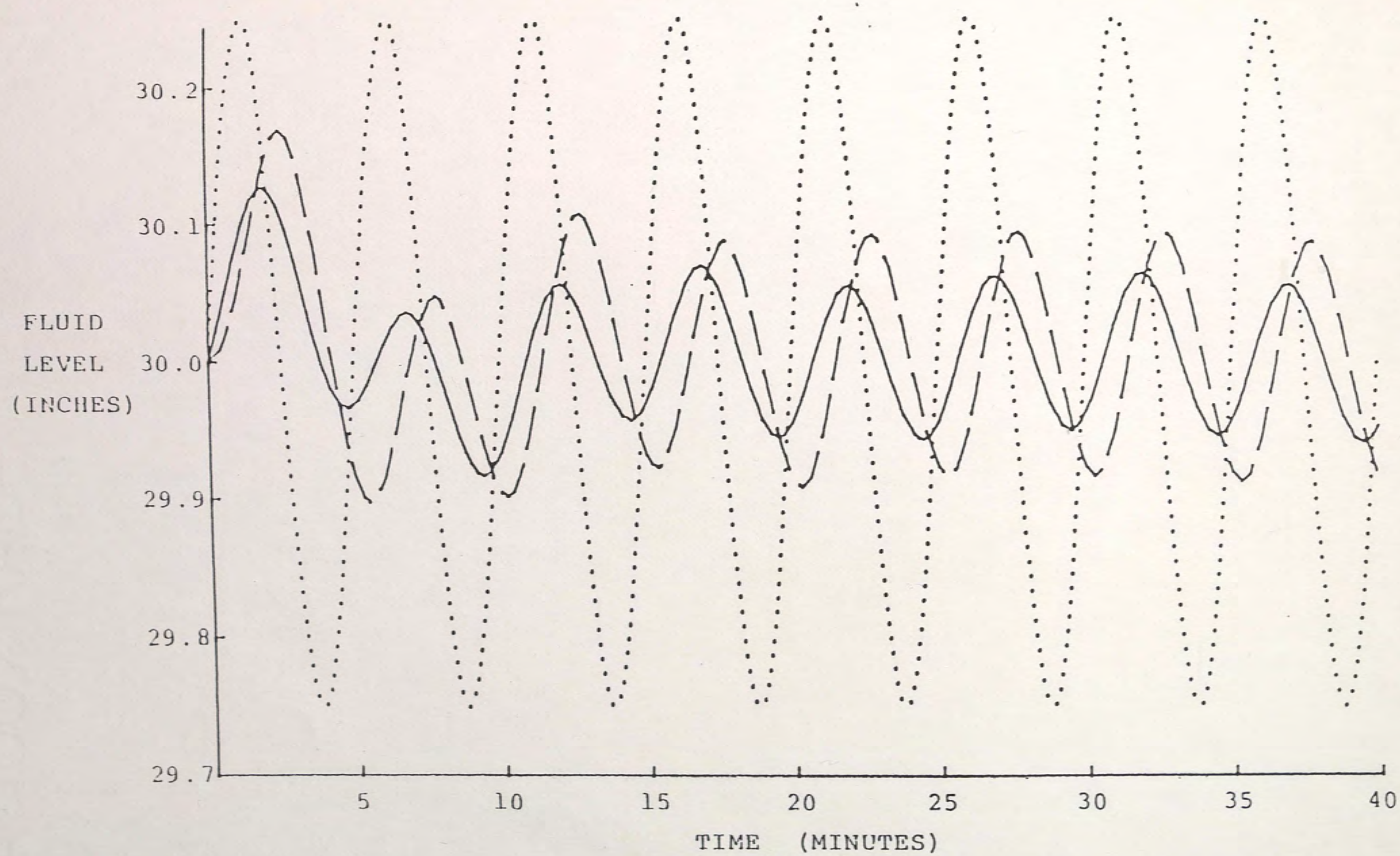


Figure 5. Level Responses to a Sine Input on Setpoint Pressure. Dotted Curve is a Sine Function of Fluid Level Setpoint with .25 Inches Amplitude. The Dashed Curve is the Response for $K_c=3$, $R_r=.5$, it Shows a Phase Lag of About $68^o$. The Solid Curve is the Response when $T_d$ is Changed from 0 to 1, Decreasing the Phase Lag by About $60^o$.

In Figure 6, two curves show the system response to setpoint level changes of 40 inches above and 40 inches below a base of 60 inches. The system response, under proportional control, does not exhibit a linear type of offset. When the setpoint is raised to 100 inches, the system undershoots that requirement by only .25 inches, and achieves steady state in approximately 5 minutes.

When the setpoint is dropped to 20 inches, the system reaches a steady state of 23.23 inches after about 100 minutes. Thus for equal setpoint change the offset is 13 times greater, and the time to equilibrium is 20 times longer for a decrease in setpoint as compared to an equivalent increase.

The response when integral control is added is shown in Figure 7. The long time to drain the tank, compared to filling, results in a large value for integral of the error, hence the system undershoots the 20 inch setpoint by 19 inches, compared to an overshoot of only 4 inches when the setpoint is raised 40 inches to the 100 inch level. Steady state at 20 inches is reached after approximately 260 minutes; steady state at 100 inches is reached after approximately 13 minutes.

Figures 8 and 9 show the system response under regulator control for outlet opening step changes from 50% to 90% and 50% to 10% opened. Figure 8 shows the proportional-integral control cases. Under proportional control

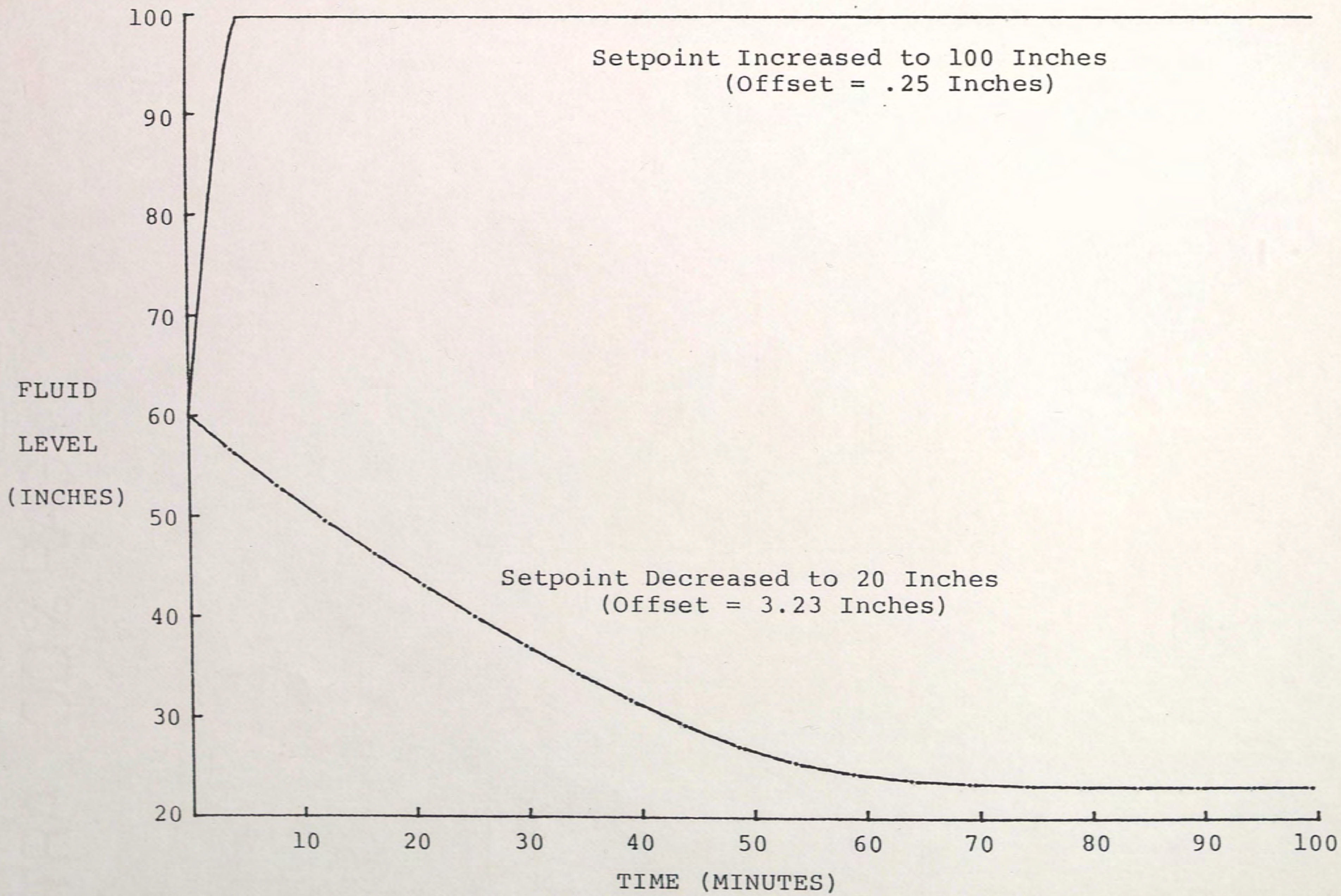


Figure 6. Proportional Control Level Responses to Setpoint Step Changes of Plus and Minus 40 Inches ($K_C$=3).

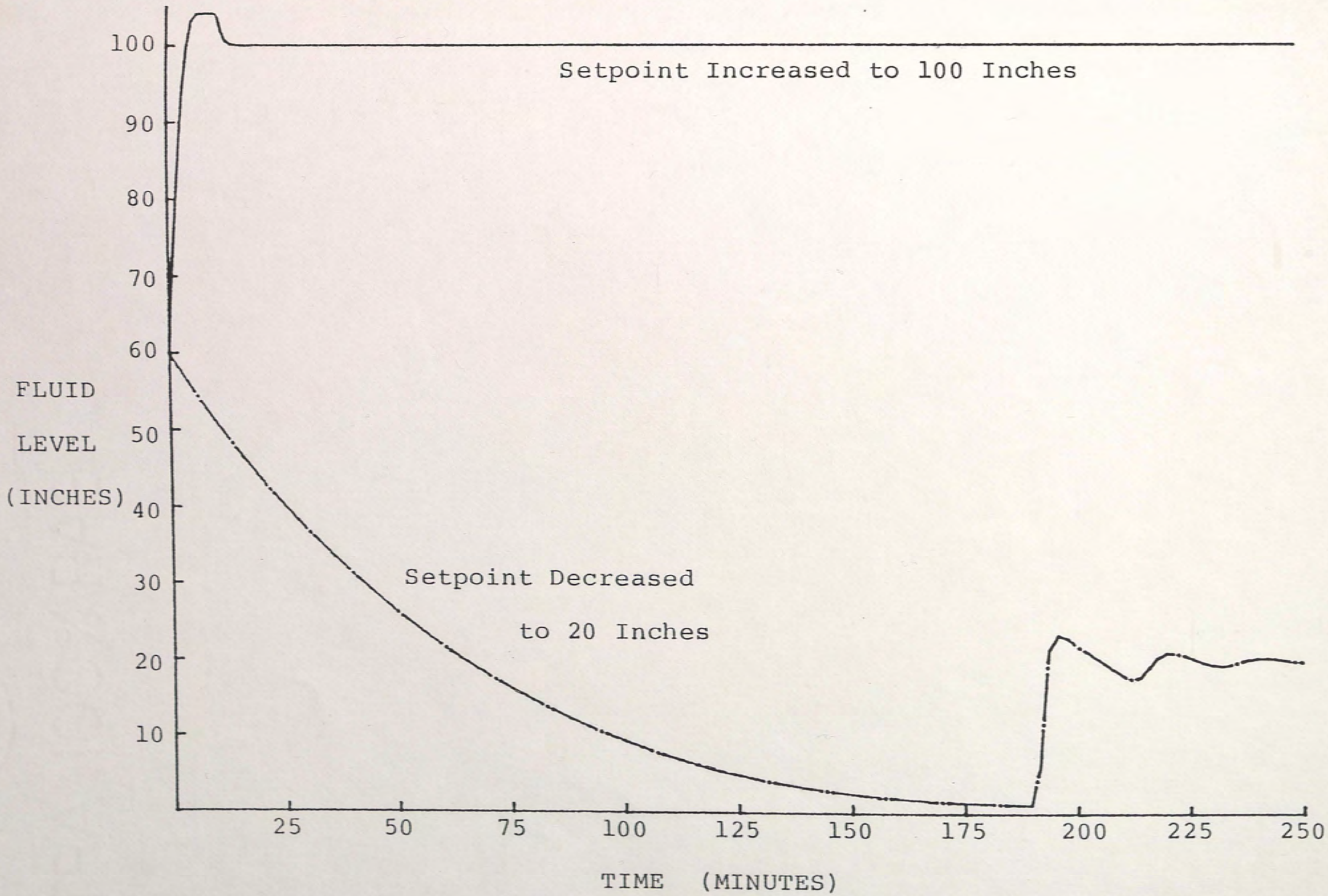


Figure 7. Proportional -Integral Control Level Responses to Setpoint Step Changes of Plus and Minus 40 Inches ($K_c$=3, $R_r$=.5).

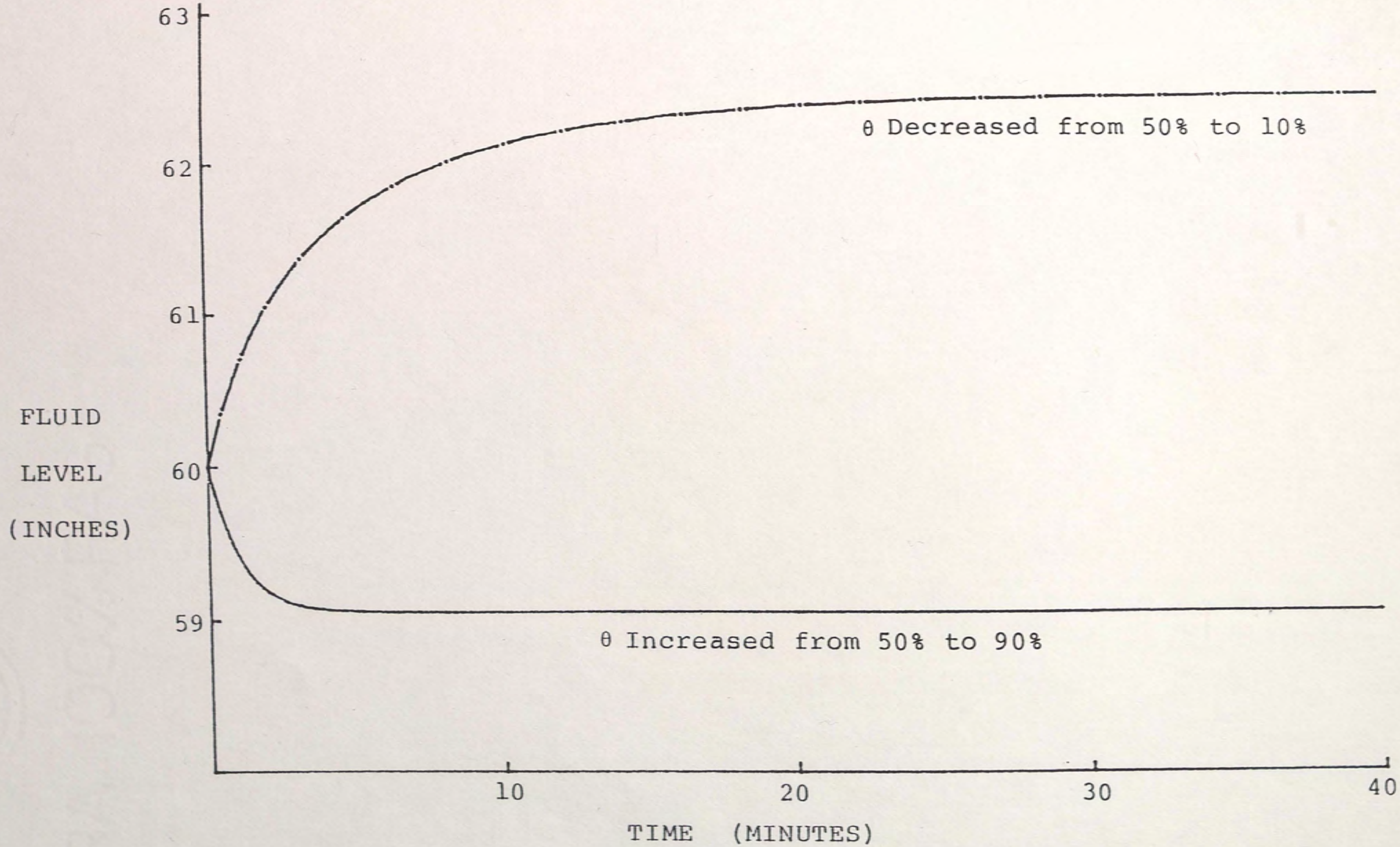


Figure 8. Proportional Control Level Responses to Outlet Opening Step Changes ($K_c$=3).

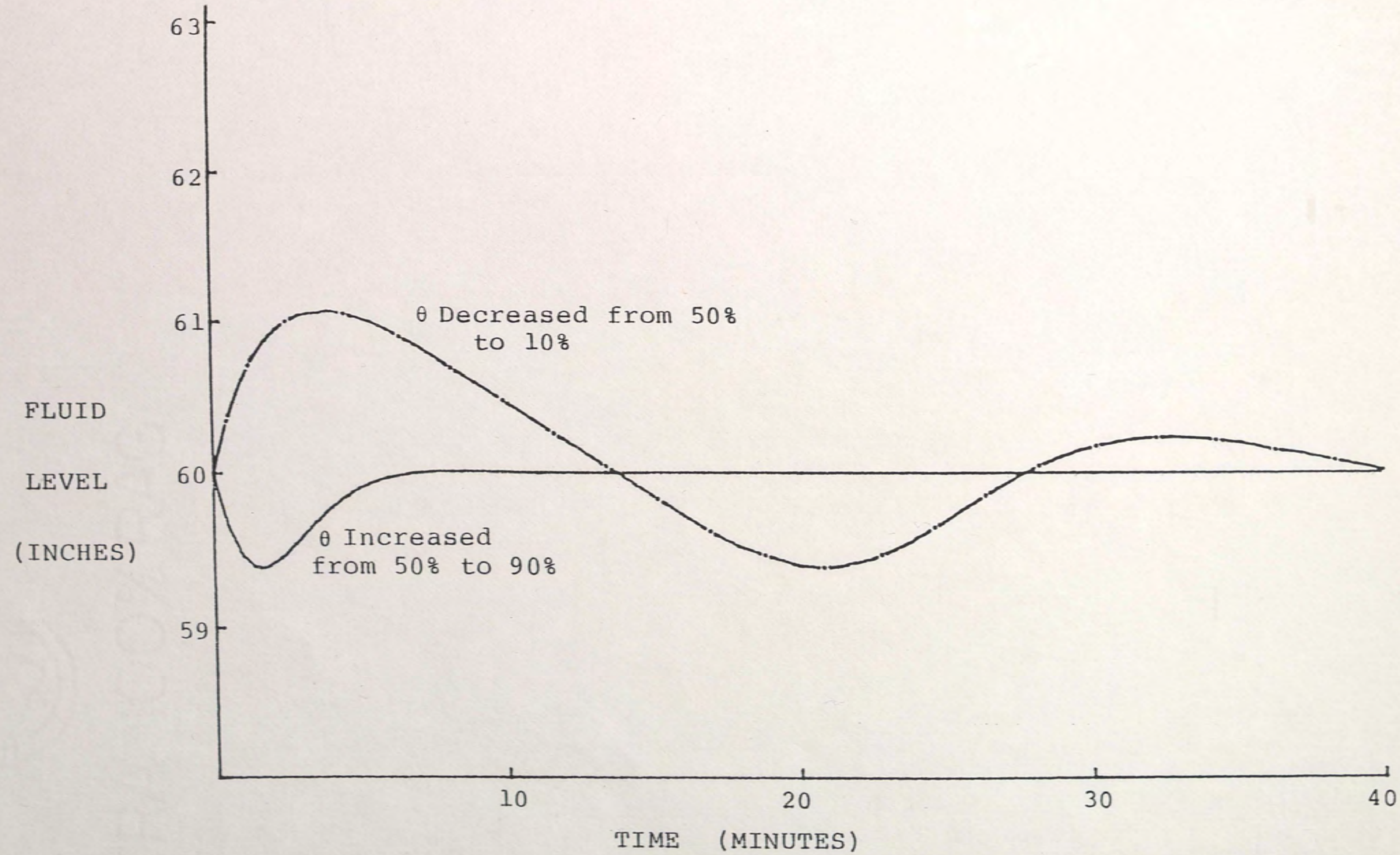


Figure 9. Level Response When Integral Control is Added to the Case Shown in Figure 8 ($K_c$=3, $R_r$=.5).

the step increase in outlet opening results in a drop in level of approximately 1 inch and steady state is reached after about 3 minutes. The step decrease in outlet opening causes a steady state increase in level of approximately 2.5 inches after about 40 minutes. These results are in accord with the behavior seen in Figure 6. That is, the system can provide better control when the forcing function calls for an increase in tank feed rather than for a function which calls for a decrease in feed rate.

The effect of adding integral control is shown in Figure 9. Again we see that an increase in outlet opening (calling for a feed flow increase) is responded to faster than a decrease in outlet opening. As expected for proportional-integral control the level returns to the setpoint of 60 inches.

Figure 10 shows the system response to inlet feed step changes of plus and minus 25 psi. Proportional and proportional-integral control examples are shown. Under proportional control the offset is .2 inches for the step increase in feed pressure and -.28 inches for the step decrease. Under PI control the system is shown to return to equilibrium faster in response to the step increase.

The simulations in figures 6 through 10 show that within the defined limits of operation, the system is very insensitive to changes in feed pressure relative to its

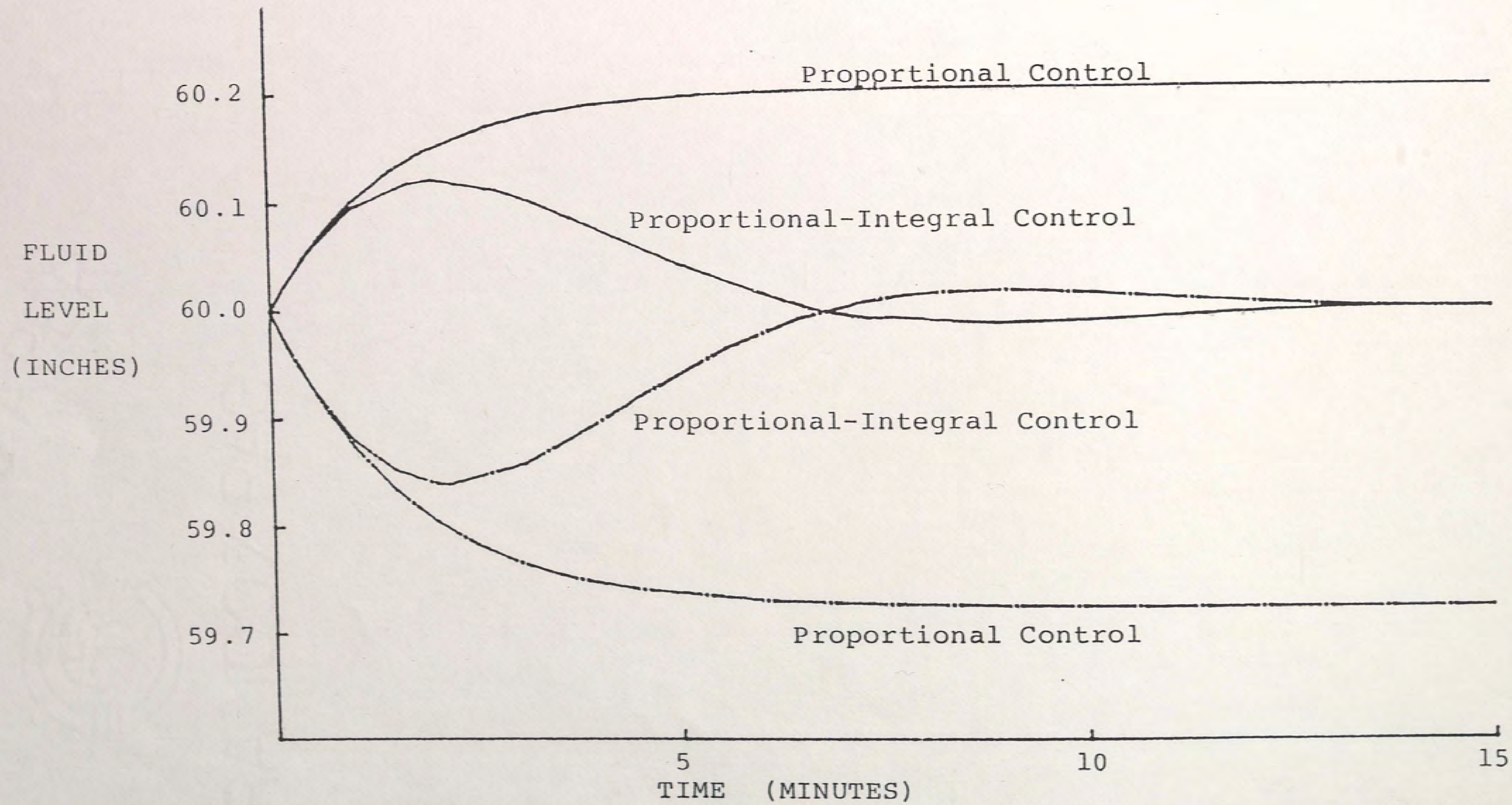


Figure 10. Level Responses to Plus and Minus 25-psi Step Changes to Feed Pressure. Solid Lines are the Responses to a Pressure Increase and Dashed Lines are Responses to a Decrease ($K_c$=3, $R_r$=.5).

response to changes in setpoint and outlet opening. In addition, due to the nonlinearities in the system, its response is much better for disturbances which are compensated for by opening the control valve.

In the next set of examples the behavior of the system for pulse inputs to the setpoint and outlet is simulated. Proportional and proportional-integral control only are modeled since derivative control works poorly when sudden changes are imposed on the system. In these examples the pulse begins 1 minute after the simulation starts, and ends 4 minutes later. The first examples, figures 11, 12, and 13 show the response to a 4-inch pulse on fluid level setpoint. Figures 14 and 15 show the response to plus and minus 40-inch pulses on level setpoint. In these examples the effect of the "limiting" nonlinearity (the control valve function) upon the system response is demonstrated and we also see the expected result that the system is temporarily disturbed, then returns to the initial equilibrium state.

Referring to Figure 11, a 4-inch pulse on the setpoint initially causes the error to go to 2.1 psi. With a proportional gain of 3 this results in a call for a control pressure, $\overline{P}_V + \Delta P_V = 9 + 3(2.1) = 15.3$. Thus the inlet valve is driven to saturation, but this condition lasts for only two seconds since during that time the wide open valve raises the level to where the error drops below 2 psi.

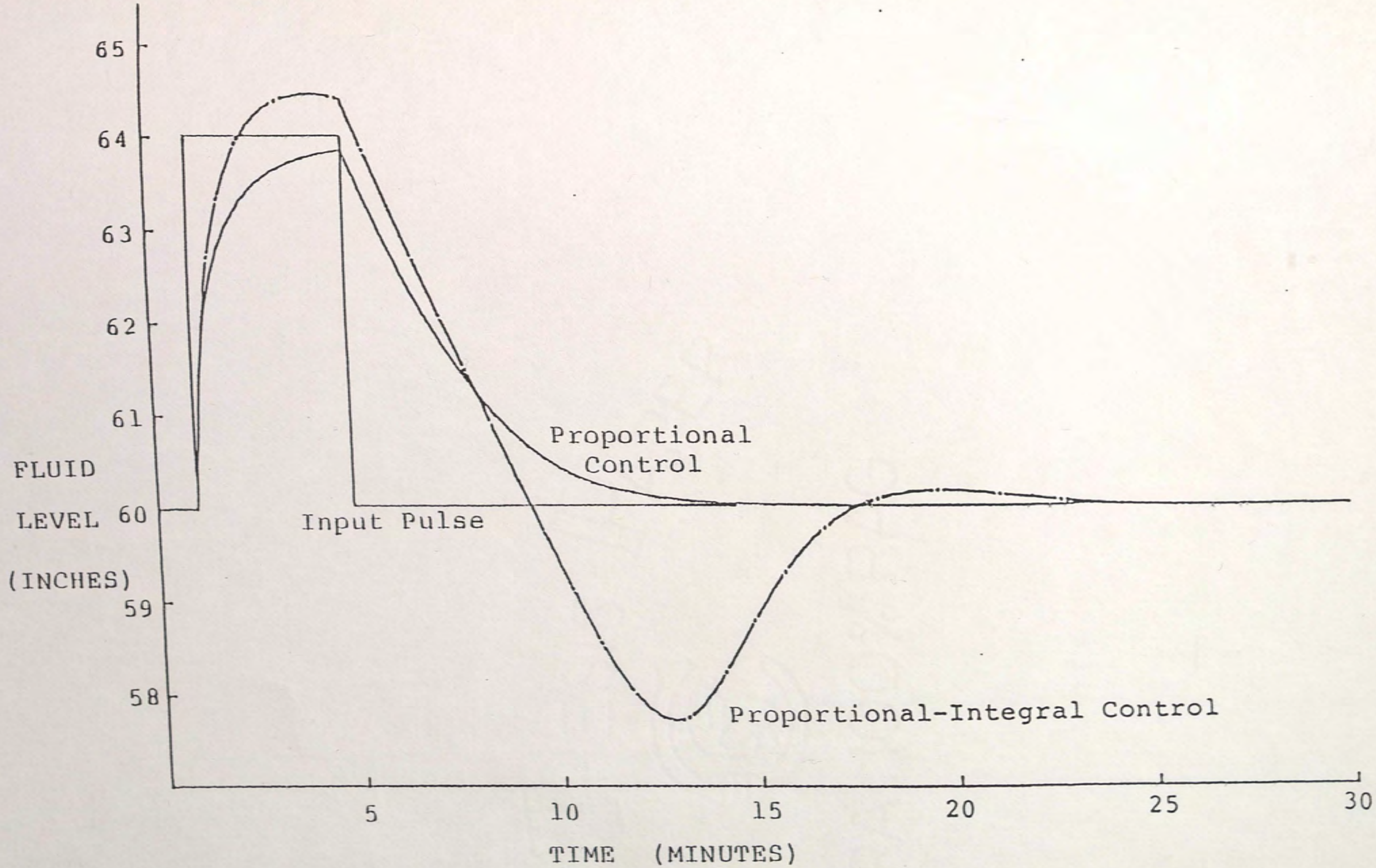


Figure 11. Level Response to a 4-Inch Pulse on Setpoint ($K_c=3$, $R_r=.5$).

In the case of proportional control the system fills up during the pulse and appears to be approaching an offset of about .15 inches when the pulse ends. At pulse end, when the setpoint drops back to 60 inches, the error is -2.006 psi and the valve is driven to saturation in the closed position. This condition lasts only momentarily, as the draining tank loses pressure with the inlet valve closed, and then within a second or two the magnitude of the error is less than 2 psi and the control pressure rises above 3 psi, opening the valve. The system then returns to the 60 inch setpoint with no offset, lagging the setpoint change by about 8 minutes.

The response curve under proportional-integral control overshoots the setpoint by approximately .4 inches, then when the pulse ends, the error is larger than in the proportional case. The valve is then driven to the closed state, but since the system drains slowly the integral error term is negative and its magnitude grows. Thus the integral error term keeps the valve in a saturated (closed) position for 5 minutes and the system undershoots the setpoint by 2.3 inches. The proportional-integral response is oscillatory and it takes 9 minutes longer than the proportional control case to return to equilibrium. These results suggest that the system under servo control would perform best with high proportional gain and no integral control for short pulses.

Figure 12 shows the effect of increasing the proportional gain to 9. Also shown is the pressure to the control valve. The tank level rises much faster with high proportional gain and stays closer to the pulse than in the case where $K_c = 3$. However, at the end of the pulse the system cannot return rapidly to the initial level. Analysis of the control pressure curve indicates that further increase in $K_c$ would not change system response during the first 15 seconds of the pulse since the inlet valve is fully open. Figure 13 shows the system response for $K_c$ values of 3, 9, and 27 in greater detail for the region near the beginning of the pulse. As expected an increase in $K_c$ cannot affect the system performance when the valve is saturated, hence the response curves for $K_c = 9$ and $K_c = 27$ are identical through 1 minute and 15 seconds, but for the highest gain the valve remains fully open for an additional 4 seconds. The increase in gain shows the greatest effect at 1 minute and 38 seconds when the level is .41 inches closer to the setpoint than in the $K_c = 9$ case. The responses are nearly identical towards the end of the pulse with only .02 inches improvement for $K_c = 27$ at 5 minutes.

In sum it is found that for short pulses integral control does not appear to be useful. It is also demonstrated that the system response to a step change cannot be arbitrarily improved by increasing the gain, due to the

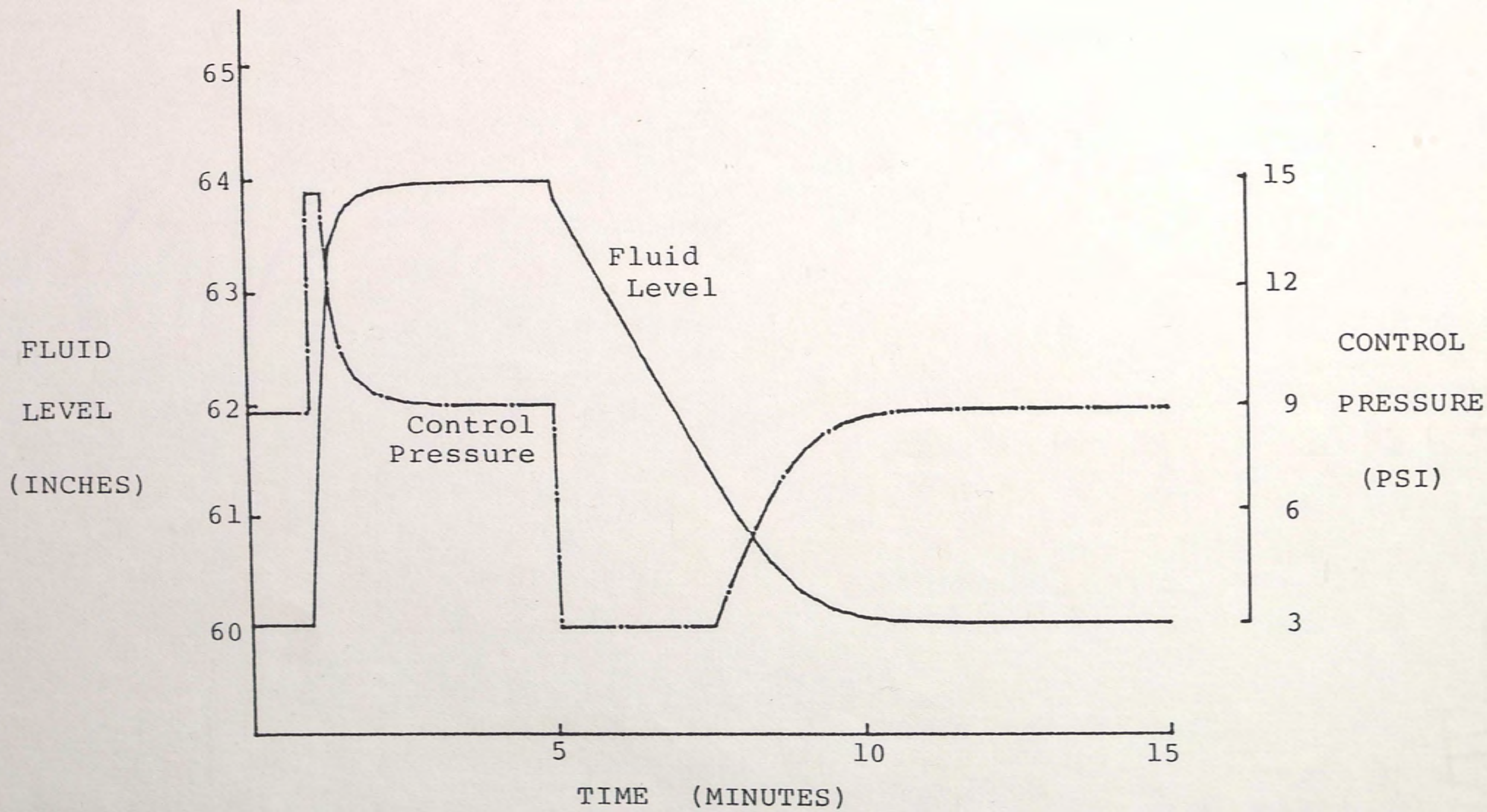


Figure 12. Level Response and Control Pressure for a 4-Inch Pulse on Setpoint Level ($K_c$=9).

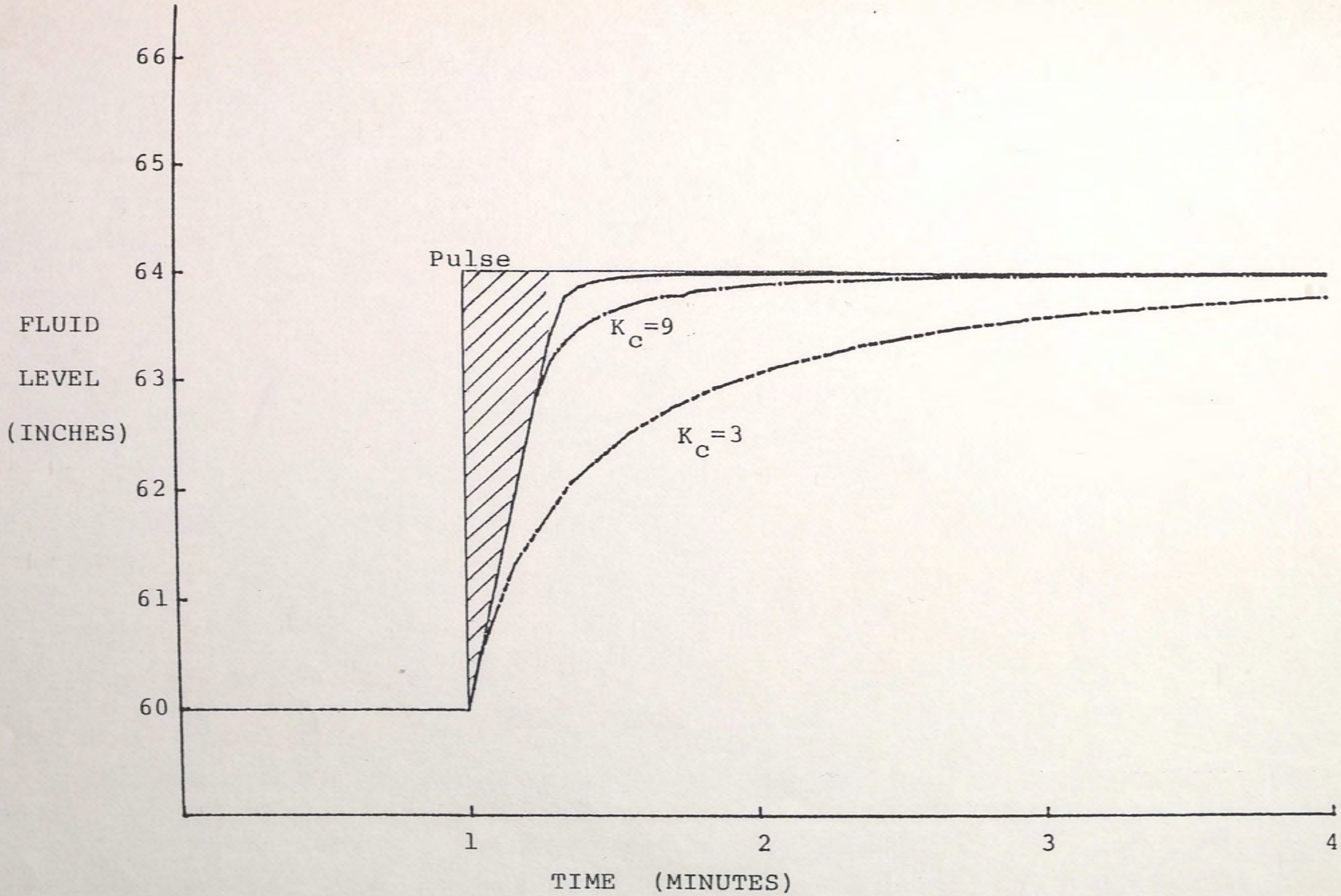


Figure 13. Level Response as $K_c$ is Increased, Showing Detail at the Leading Edge of the Input Pulse. The Shaded Area is the Forbidden Region. The Highest Curve is the Response When $K_c$=27.

limiting type of nonlinearity on the inlet valve. Comparing the level responses in Figure 13, when the gain is increased from 3 to 9 and 9 to 27 the diminishing returns for increasing $K_c$ are shown. The limits on the inlet valve action define a forbidden region of response through 1 minute and 19 seconds. That is, regardless of gain, the system response cannot improve. The forbidden region can be defined beyond 1 minute and 19 seconds using arbitrarily large values of $K_c$.

Figure 14 shows the response for pulses of plus and minus 40 inches and the corresponding forbidden regions. These large jumps in setpoint cause the inlet valve to remain fully on throughout the pulse for the positive pulse and fully closed for the negative pulse. Thus the valve is at its limits in each case and the forbidden region is completely determined from the results of the $K_c = 3$ case. The forbidden region is larger for the negative pulse because the system cannot respond as quickly to calls for a drop in level. At the end of the positive pulse the valve shuts off and remains off for 25 minutes defining a large forbidden region. At the end of the negative pulse however, the system has not drained sufficiently to result in an error exceeding the limits, and no forbidden region can be defined from the $K_c = 3$ simulation results.

Figure 15 shows the effect of adding integral control. Compared to Figure 14, integral control shows no improvement

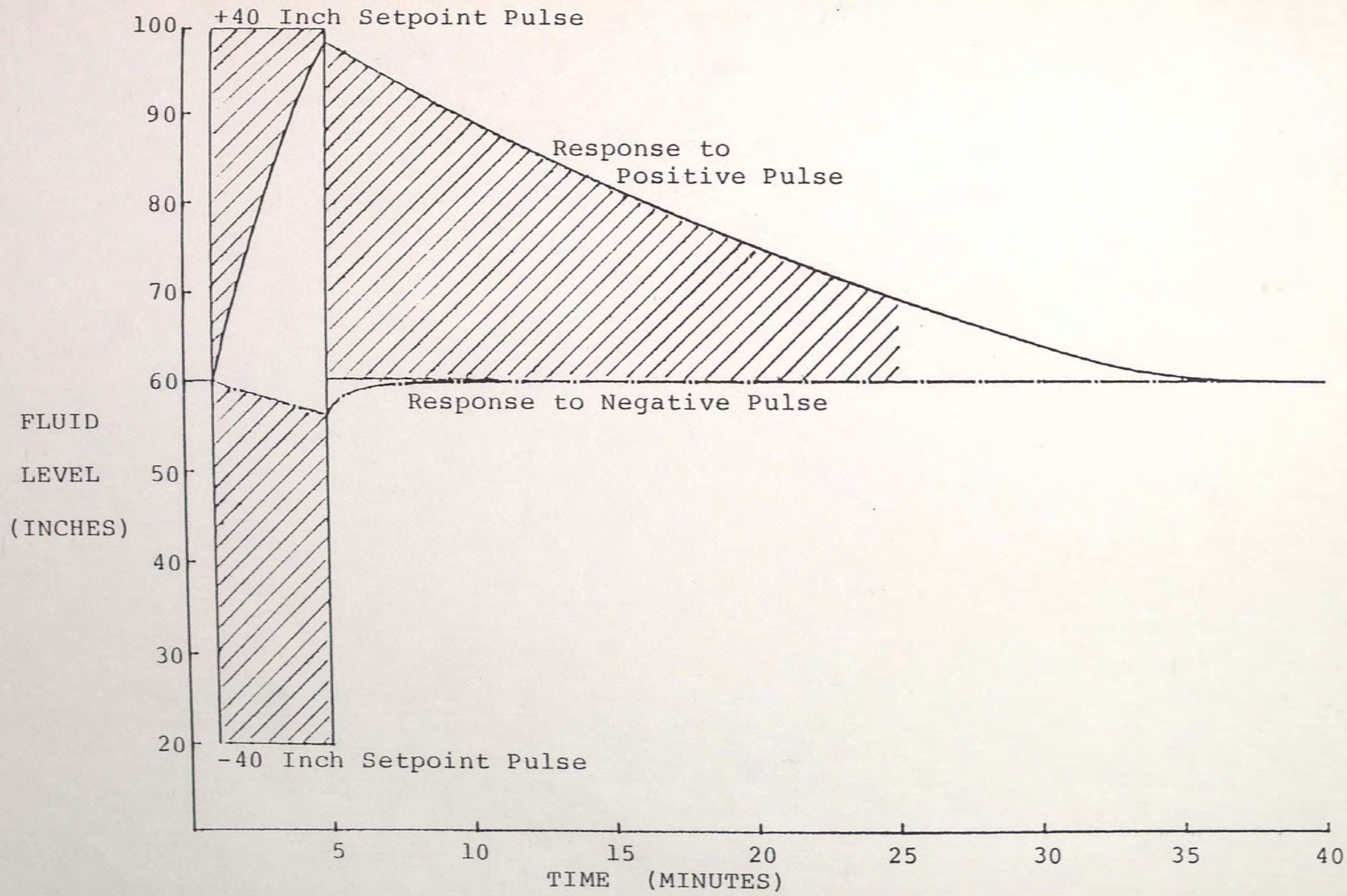


Figure 14. Level Responses to Setpoint Pulses of Plus and Minus 40 Inches. Forbidden Regions are Shaded ($K_C$=3).

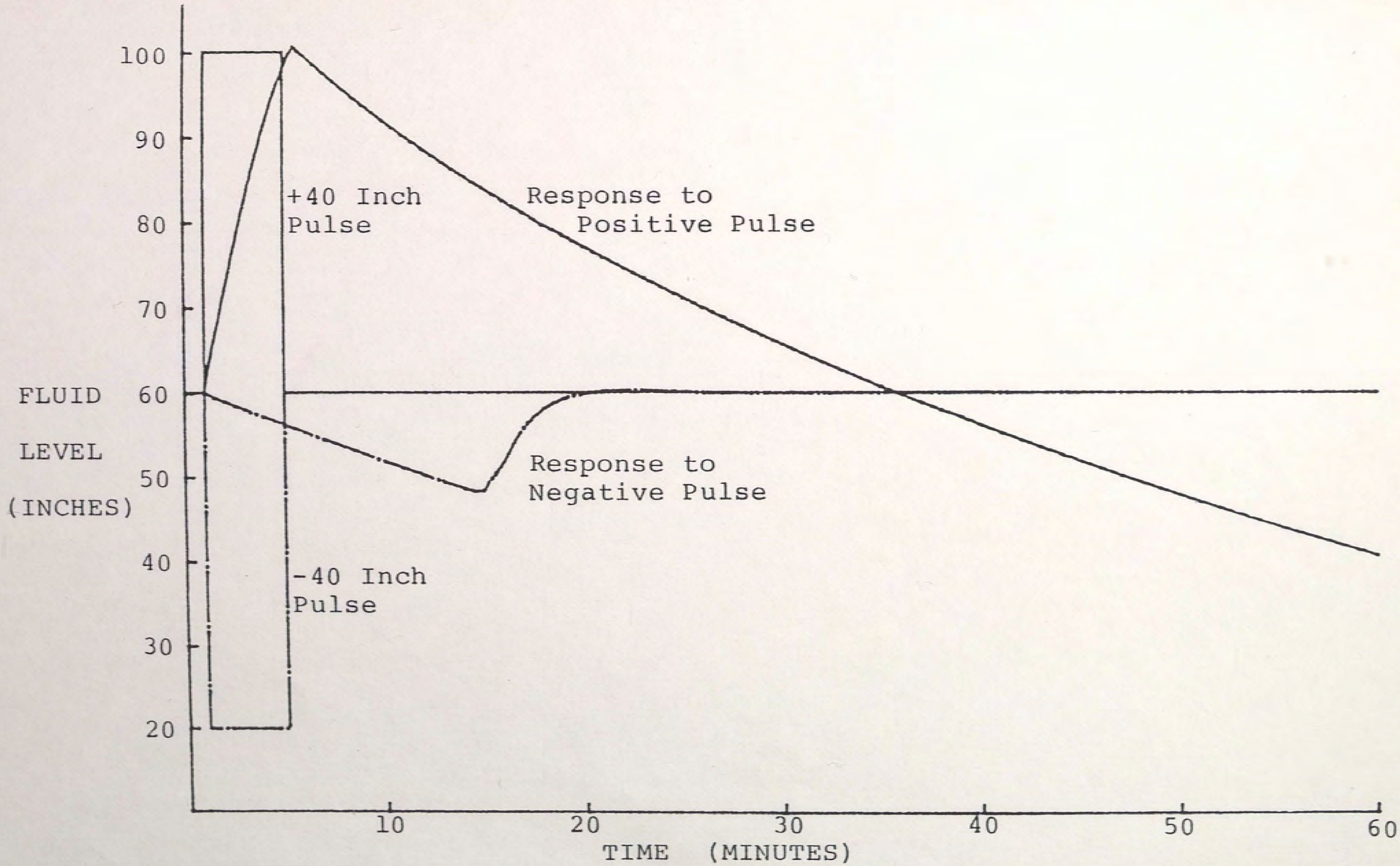


Figure 15. Proportional-Integral Control Level Responses to Setpoint Pulses of Plus and Minus 40 Inches ($K_C$=3, $R_r$=.5).

during the pulse and a decrease in performance following it. In effect, adding a factor for the accumulated error only decreases the performance when the system is already operating at its design limits.

Whereas it was found under servo control that adding integral control to proportional appeared to be a poor choice for pulse input, it is shown in the next example (Figure 16) that for the case of regulator control it can improve the performance. This is largely due to the fact that under proportional control the system, with the parameter limits specified, will rarely be driven to the control valve limits when subject to a regulator type disturbance. Figure 16 shows the response to a pulse change from $\theta$ = 10% to 90% and then back to 10% over a 4-minute period starting at 1 minute after the beginning of the simulation. Adding integral control with a reset rate of .5 decreases the maximum setpoint deviation by about 3 inches near the end of the pulse, nearly eliminating the offset shown for proportional control. After the pulse the two control modes show comparable setpoint deviations. Under proportional control the system slowly returns to the setpoint from below and under integral control the system overshoots the setpoint and remains above it as the integral error term slowly decreases allowing the system to return to the initial steady state in the expected oscillatory fashion.

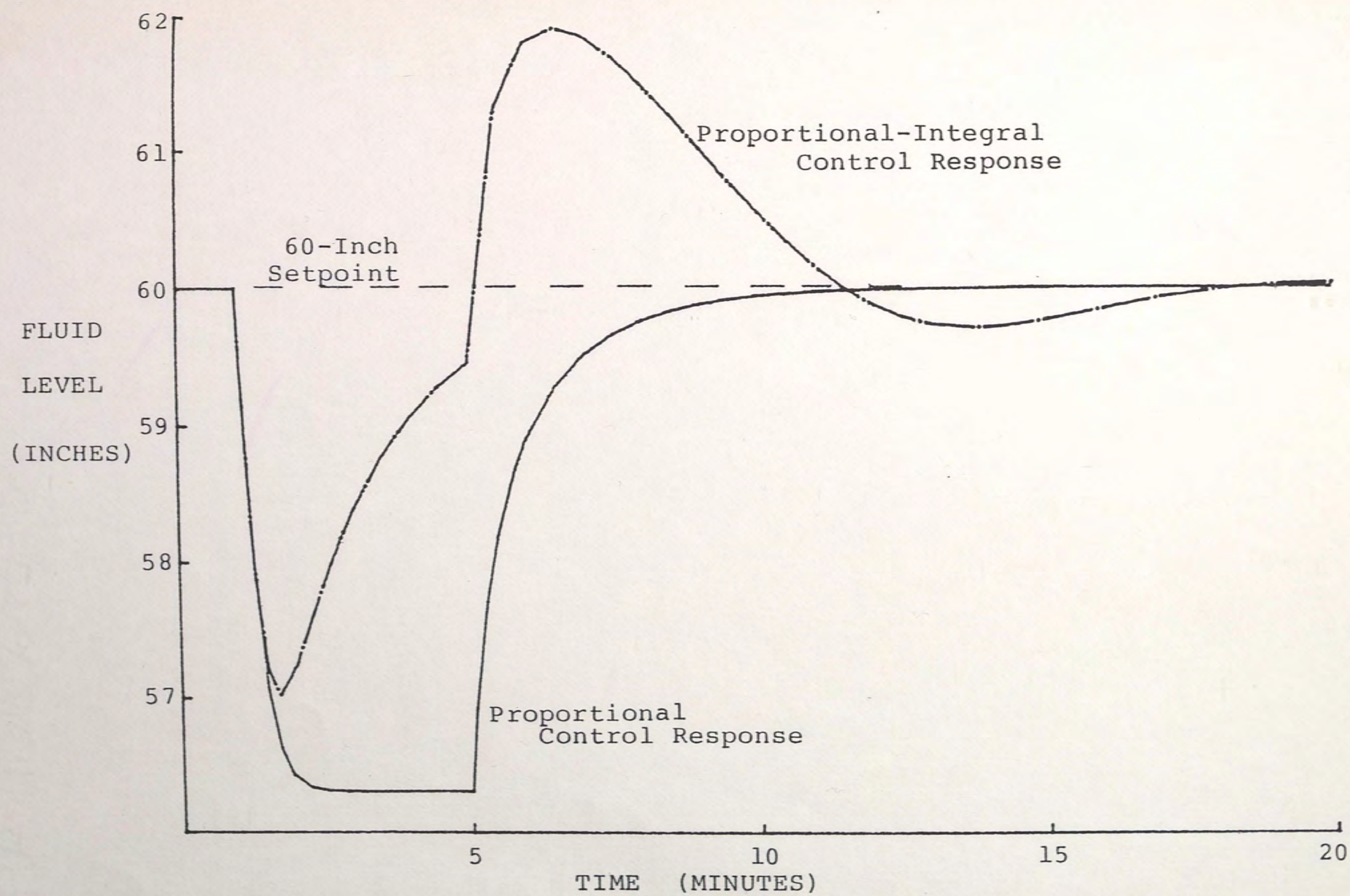


Figure 16. Level Responses to a 4-Minute Pulse on the Outlet Valve Opening. At 1 Minute the Valve Changes from 10% to 90%, Then Returns to 10% at 5 Minutes ($K_c$=3, $R_r$=.5).

The last type of input function to be considered is the ramp input. In the simulations that follow the effect of varying the amount of derivative control is demonstrated. The base case is a ramp increase in $\theta$ from 10% to 90% in 1 minute. The proportional gain is 3 and the reset rate is .5. The derivative time, $T_d$, is varied and the level responses for cases where $T_d$ is zero, .5, 1.5 and 3 are shown in figures 17 and 18. As expected, an increase in $T_d$ causes the system to respond to the ramp faster as demonstrated by the response curves for the first minute of simulation. At one minute the $T_d$ = 3 case shows only .69 inches deviation from the setpoint while the $T_d$ = 0 case is 2.56 inches below it. During the ramp the increasing $\theta$ causes the derivative of the error to increase, however after the plateau is reached $\theta$ is held constant and the derivative of the error decreases. Then integral control becomes the dominant factor. When the system reaches the point where the inlet flow equals the outlet flow (the first minimum on each response curve in Figure 18) the system begins to climb back toward the setpoint. Here the drawback of derivative control is demonstrated, since it slows down the rate at which the system returns to the setpoint. Then the integral of the error accumulates over a longer period of time. There are two competing factors then:

1. increasing the derivative time decreases the error at the end of the ramp and

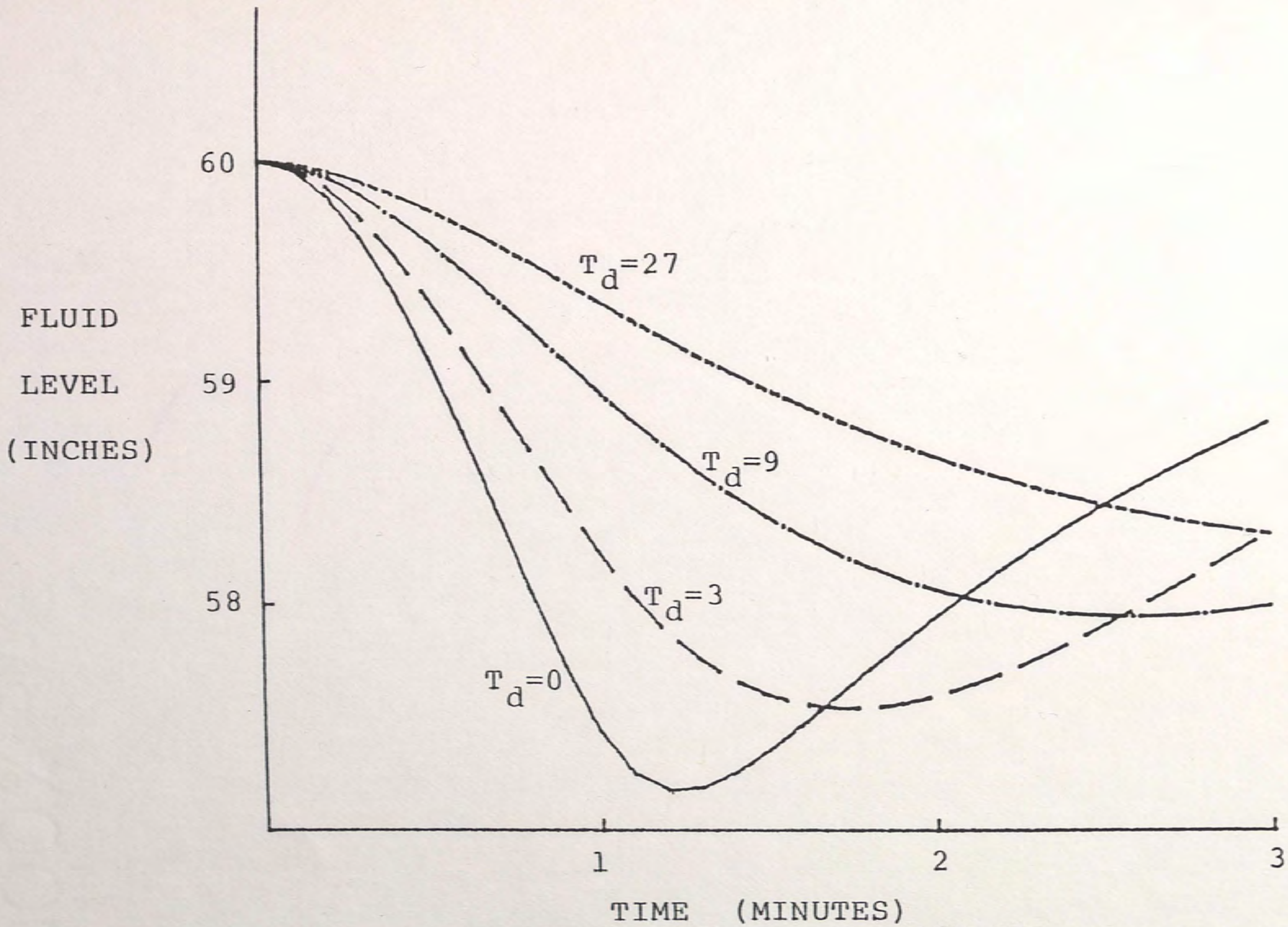


Figure 17. Proportional-Integral-Derivative Control Responses to a Ramp Increase in Outlet Opening from 10% to 90% 1 Minute After the Start of the Simulation. As $T_d$ is increased the System is Better Able to Maintain the Setpoint During the Ramp ($K_c$=3, $R_r$=.5).

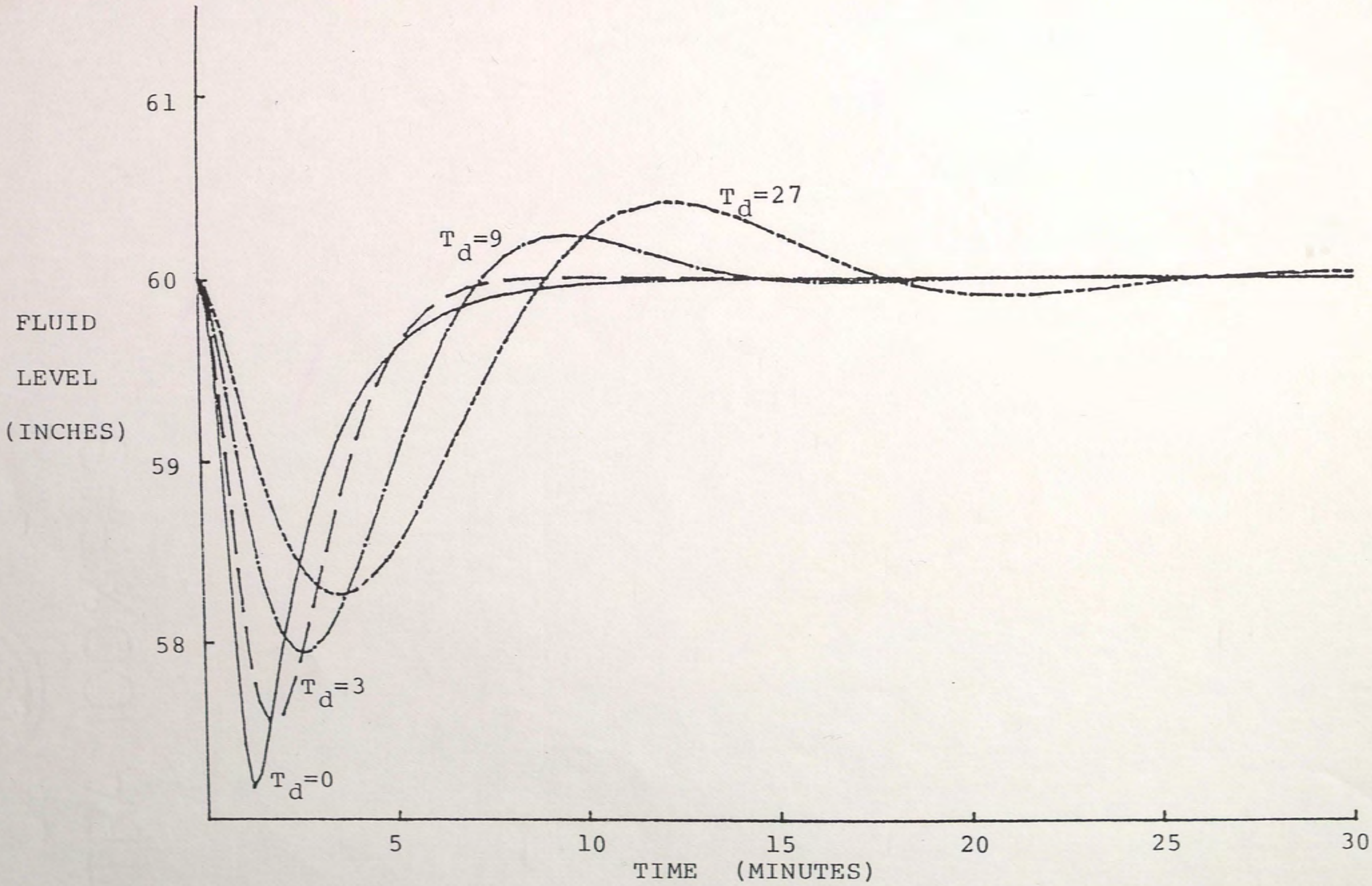


Figure 18. Level Responses Shown for Later Times Under the Same Conditions as the Figure 17 Cases.

2. increasing the derivative time prolongs the time it takes to return to the setpoint and consequently boosts the error integral and tends to <u>increase</u> the wavelength and amplitude in the transient that follows.

# CHAPTER 5 COMPARISON TO TUTSIM

TUTSIM is a generalized modeling language designed for use in simulation of dynamic systems. A given differential equation is modeled by converting all functions, arithmetic operations, differentiations and integrations in the equation to an operator form using the notation of the modeling language (Applied i, 1984). The TUTSIM language allows for direct translation of the system Equation 14 for solution. However, TUTSIM uses a first-order differentiator which was found to be very unstable for the cases tested, thus limiting TUTSIM's use to proportional and proportional-integral control. The TUTSIM integrator is a fourth-order Adams-Bashforth type, but the starting method is not specified. A description of the TUTSIM model corresponding to Equation 14 is listed in Appendix II. It was found that for every proportional and proportional-integral control case tested (15 cases in all) the TUTSIM model and the model developed herein showed excellent agreement. There was typically less than a .1% difference in fluid level, control valve pressure, inlet flow and outlet flow from the start of simulation to equilibrium. The comparisons included cases under servo and regulator control for step, ramp, sine and exponential inputs using various values of $K_c$ and $R_r$. The time step was one second for all 15 cases. In the sections that follow a detailed comparison of simulation results from the two models is presented, followed by a discussion of the unsuccessful ideal PID model and approximation to an ideal PID model using TUTSIM.

The scheme for comparing the two models is to choose a case where the system response is near the limits of design. Servo control for a step change in setpoint was chosen for this purpose since the system is most sensitive to this type of input. Two servo cases, step increases of 3 inches and 10 inches above a base of 60 inches, were chosen since the 3 inch case does not drive the inlet valve to saturation while in the 10 inch case the valve is driven fully open and remains open for approximately 50 seconds.

It is also desired to test the effect of using a simple trapezoidal rule in NONLINRK to model integral control, so the comparison case must include integral control. NONLINRK uses a fourth-order technique to solve Equation 14 as does TUTSIM, however the integral approximation and the linear approximation to the exponential in NONLINRK contribute errors of unknown order on the controlled variable, pressure in the tank. The results of several simulation trials indicate that these approximations in NONLINRK have no significant affect, however the cases mentioned above used a 1-second time step, and the influence of time step upon convergence must be examined to compare the models.

Tables III and IV show how TUTSIM and NONLINRK simulations diverge as the time step is increased. In Table III the system is responding under servo control to a setpoint increase of 3 inches and in Table IV the increase is 10 inches. The Table III results show that the models agree in predictions of fluid level to 5 significant figures for a 1-second time step. When the time step is increased to 30 seconds TUTSIM deviates 1.1 inches and NONLINRK .7 inches from the result shown for the 1-second time step. The results for the later times show slightly better convergence for NONLINRK, however the differences are insignificant considering that the time step was raised by a factor of 30.

In Table IV the model results are compared for time steps of .1 second, 1 second, 6 seconds, 12 seconds and 30 seconds. In this table the models converge to 5 significant figures for time steps of .1 second and 1 second. With an increase to a 6-second step NONLINRK shows no significant deviation through 48 seconds, and TUTSIM shows a deviation of .02 inches. At one minute, however, the TUTSIM deviation is .052 inches too low and the NONLINRK result is .187 inches too high. After one minute the deviation decreases rapidly for NONLINRK and there is no significant difference between the models. As the time step is increased to 12 and then 30 seconds

TABLE III

TUTSIM vs. NONLINRK MODEL RESPONSES UNDER PROPORTIONAL-INTEGRAL CONTROL FOR A STEP CHANGE IN SETPOINT FROM 60 INCHES TO 63 INCHES

| | FLUID LEVEL IN INCHES | | |
|---|---|---|---|
| TIME | 1 SECOND TIME STEP | 30 SECOND TIME STEP | |
| (MINUTES) | BOTH MODELS* | TUTSIM | NONLINRK |
| 0.0 | 60.000 | 60.000 | 60.000 |
| .5 | 61.768 | 62.900 | 61.064 |
| 1.0 | 62.546 | 62.027 | 62.133 |
| 1.5 | 62.970 | 62.955 | 62.877 |
| 3.0 | 63.381 | 63.297 | 63.578 |
| 5.0 | 63.261 | 63.239 | 63.410 |

*TUTSIM and NONLINRK agree to 5 significant figures for a 1-second time step.

## TABLE IV

DEVIATION OF TUTSIM AND NONLINRK RESULTS AS THE TIME STEP IS INCREASED FROM .1 SECOND TO 30 SECONDS. THE SYSTEM IS UNDER PROPORTIONAL-INTEGRAL CONTROL ($K_c$=3, $R_r$=.5) RESPONDING TO A STEP CHANGE IN LEVEL FROM 60 TO 70 INCHES.

| TIME MINUTES | .1 SECOND TIME STEP RESULTS INCHES | INCHES DEVIATION FROM .1-SECOND TIME STEP RESULTS | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | | LENGTH OF TIME STEP | | | | | | | |
| | | 1 SECOND | | 6 SECONDS | | 12 SECONDS | | 30 SECONDS | |
| | | TUTSIM | NONLINRK | TUTSIM | NONLINRK | TUTSIM | NONLINRK | TUTSIM | NONLINRK |
| 0.0 | 60.000 | N.C.* | N.C. | N.C. | N.C. | N.C. | N.C. | N.C. | N.C. |
| 0.4 | 64.559 | -.001 | N.C. | -.021 | N.C. | -.079 | N.C. | - | - |
| 0.5 | 65.682 | N.C. | N.C. | -.021 | N.C. | - | - | -.425 | N.C. |
| 0.8 | 69.009 | N.C. | N.C. | -.023 | .001 | -.017 | N.C. | - | - |
| 1.0 | 70.224 | -.001 | .001 | -.052 | .187 | .872 | .965 | .457 | .965 |
| 1.5 | 71.017 | N.C. | N.C. | -.021 | .031 | - | - | .698 | .260 |
| 2.0 | 71.168 | N.C. | N.C. | -.011 | .003 | -.050 | .067 | -.677 | .038 |
| 4.0 | 70.644 | N.C. | .001 | .004 | .023 | -.004 | .135 | -.017 | -.137 |
| 8.0 | 69.951 | -.001 | N.C. | .001 | N.C. | .007 | .010 | .038 | -.009 |

*N.C.- No change, deviation is less than .001 inches.

the deviations grow but not in any particular pattern, and no significant advantage is shown for either model.

A study of Table IV shows one interesting effect of the NONLINRK linearization of the exponential function. Note that for times less than .8 minutes, NONLINRK results are in perfect accord for all time steps. This is because in the most exact case there is no inlet valve action through .8 minutes, but between .8 minutes and .9 minutes the inlet valve begins to close. Now in NONLINRK if the inlet valve is fully open at the beginning of the time step then the system equation is of the form for a fully open valve throughout the following interval. This is just fine if the system should be open, however if a simulation using a smaller time step indicates that the valve should close during the interval then NONLINRK computes an artificially high level. This artificially high level is shown in the results at one minute. Also note that the deviation is .965 inches for a 12 second and a 30 second time step. This is because in both cases the valve is saturated until one minute, and the valve error is maintained at the same rate for the same length of time.

In sum, the test cases above show that for the given system the NONLINRK model with its approximations, is as accurate as a fourth-order model with no approximations. The results indicate that for convergence to within .01

inch a step size of about one second should be used, however, this is for the base case presented in this work. For example, using a tall thin tank and larger inlet valve, the system could fill and empty much more rapidly. Consequently smaller step sizes would be expected to show better convergence for a system which is configured with less stability than the base case used in the simulations presented herein.

The comparison studies above show excellent agreement between TUTSIM and NONLINRK, for all cases excluding derivative control. To investigate this type of control, two options for modeling the system using TUTSIM were explored. TUTSIM offers an operator, labelled a PID controller block, which simulates a non-ideal PID controller. According to the transfer function of this block a parameter $\alpha$, is input as a measure of the deviation from an ideal PID controller. The transfer function (Applied i 1984) is given by:

$$\frac{U(s)}{I(s)} = K_c \left[ \frac{1+sT_d}{1+s\alpha T_d} + \frac{R_r}{s} \right] \qquad [23]$$

where U and I are the controller output and input respectively. Referring to Equation 10a we find that the transfer function representation of the ideal controller in NONLINRK is given by:

$$\frac{U(s)}{I(s)} = K_c \left[ 1+ \frac{R_r}{s} + sT_d \right] \qquad [24]$$

A comparison of these transfer functions shows that for $\alpha=0$ they are identical. TUTSIM simulations with small values of $\alpha$ were performed for comparison with NONLINRK PID simulations. TUTSIM results showed severe instability, with the inlet valve jumping open then closed on alternate time steps. As $\alpha$ was increased this behavior stopped for the cases tested. No such instability was evident in the corresponding NONLINRK cases and although NONLINRK results under PID control are not confirmed, the phase shift and response to ramp input demonstrated in Chapter 4 indicate the proper trends for the addition of derivative control to proportional-integral control test cases.

The second option for constructing a TUTSIM ideal PID controller model was to model Equation 14 using integrator and differentiator blocks in place of the non-ideal PID block. Again the simulation was highly unstable. The TUTSIM test case parameters were identical to those used in the NONLINRK simulations shown in figures 17 and 18. As $T_d$ was increased from .25 to .5 to 1.5 the onset of the TUTSIM simulation instability was sooner and the shifts in amplitude were greater. Figure 19 shows the three curves for the control pressure corresponding to the $T_d$ cases above. When $T_d$ = .25 the instability starts at 67 seconds

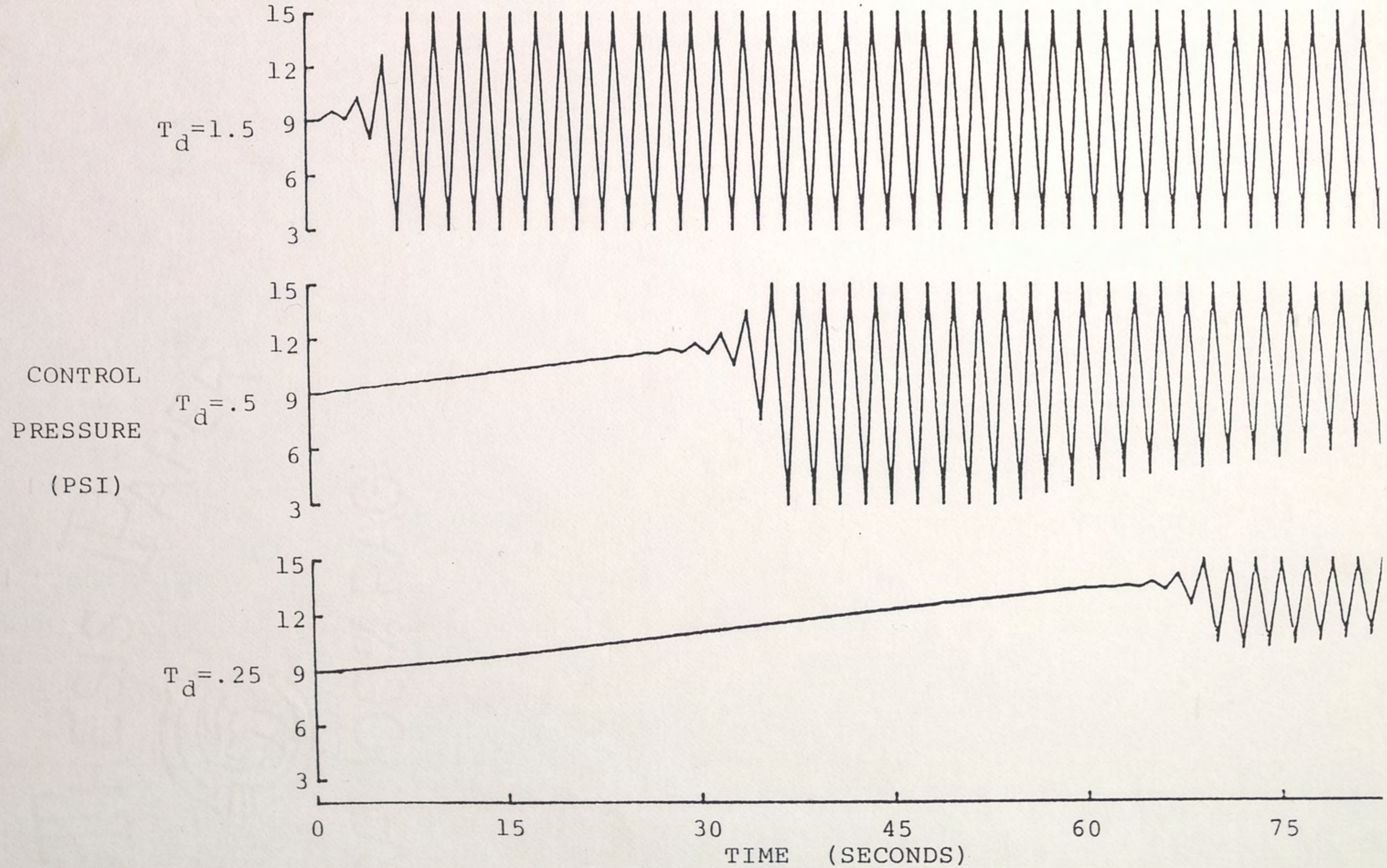


Figure 19. TUTSIM Model Results Showing the Onset of the Instability in Control Pressure as $T_d$ Increases. Ramp Outlet Valve Function Input as in Figures 17 and 18 ($K_c$=3, $R_r$=.5).

and the control pressure flip-flops from about 10 psi to 15 psi on each time step. Increasing $T_d$ to .5 causes the instability to develop earlier, at 31 seconds, and the control pressure now flip-flops across the range from 3 to 15 psi at each time step. With $T_d = 1.5$ the instability starts at 2 seconds and flip-flops from 3 to 15 psi.

Thus the two attempts to generate corroborative derivative control results from TUTSIM failed. The TUTSIM simulations are probably unreliable. TUTSIM uses a first-order differentiator to compute the derivative of P while NONLINRK uses the most recent estimate from a fourth-order technique to estimate the derivative. Also, simulations were run to test the affect of time step size and the NONLINRK derivative control results show a regular pattern of convergence, while the TUTSIM simulations did not.

# CHAPTER 6 CONCLUSIONS AND SUGGESTIONS FOR FURTHER WORK

Simulations of the nonlinear tank system using the model NONLINRK have predicted that frequency response, offset and saturation effects are nonlinear in nature. In addition the model response has been shown to approach that of the linear case for sufficiently small inputs. NONLINRK PI control simulations have been verified using the modeling language TUTSIM. The TUTSIM model uses a fourth-order Adams-Bashforth method. The NONLINRK model, which is based on Gill's fourth-order Runge-Kutta algorithm but includes two first-order approximations, did not suffer any significant loss of accuracy. Further verification of NONLINRK is needed and laboratory tests using the Technovate system (Figure 2) are recommended for future work.

To develop the model NONLINRK, a linearization of an exponential function of the dependent variable, its derivative and integral was necessary to cast the system equation in a form amenable to a numerical solution technique. (The remaining nonlinearities in the system equation were not altered.) The success of this solution suggests that in place of the usual linearization of all system equations, as is proposed in most texts on process control systems, linearize only the functions necessary to separate the highest-order derivative, and then relinearize at the beginning of each time step in a numerical solution technique.

The model worked quite well under every simulation attempted by this author. However, given a less stable system configuration to model (in terms of the base case parameters), the technique could be improved. For example in Chapter 4, after the linearization of the exponential function, the option of differentiating the system equation could be selected. Then the numerical method could proceed in two stages, using the Runge-Kutta algorithm to solve the second order nonlinear system, finding the derivative of P and $\dot{P}$ respectively. This eliminates the use of the less accurate trapezoidal rule to find the integral of P in the NONLINRK solution presented herein.

The solution technique could be made more efficient by switching from the Runge-Kutta to an Adams-Bashforth explicit fourth-order method after the first four time steps. This may decrease the amount of computation time by as much as one-half.

Another modification to the technique that may be worthwhile is to use a slightly different strategy to linearize the exponential function when its argument undergoes a significant change within a time step. Figure 20 shows a hypothetical starting point on the exponential function, corresponding to a control pressure of 8 psi. Suppose that during the next time step the system is responding to a call to increase the fluid level resulting in an increase of $P_v$ to 14 psi as computed using NONLINRK. In this case the linearization leads to a solution that has proceeded using the tangent to the exponential even though the tangent deviates significantly when $P_v$ changes this much. The tangent approximation to the exponential is 80% too low at the end of the time step. Now if a chord were carefully selected to approximate the exponential over this interval, then the approximation is improved. The chord shown in Figure 20 was chosen to pass through the exponential at the midpoint, when $P_v$ = 11 psi, of the interval found in the approximation using the tangent. Then the strategy for using the chord approximation is to choose a maximum allowable deviation in $P_v$ between time steps. If this deviation is exceeded then recompute the solution for this time step using the first result as a guide for fitting the chord.

The example in Figure 20 shows a chord that passes through the initial point and a point in the middle of the interval found in the first approximation using the tangent. This placement of the chord is arbitrary but further analysis could pursue the development of a technique that is based upon minimizing the integral of the deviation of the linear

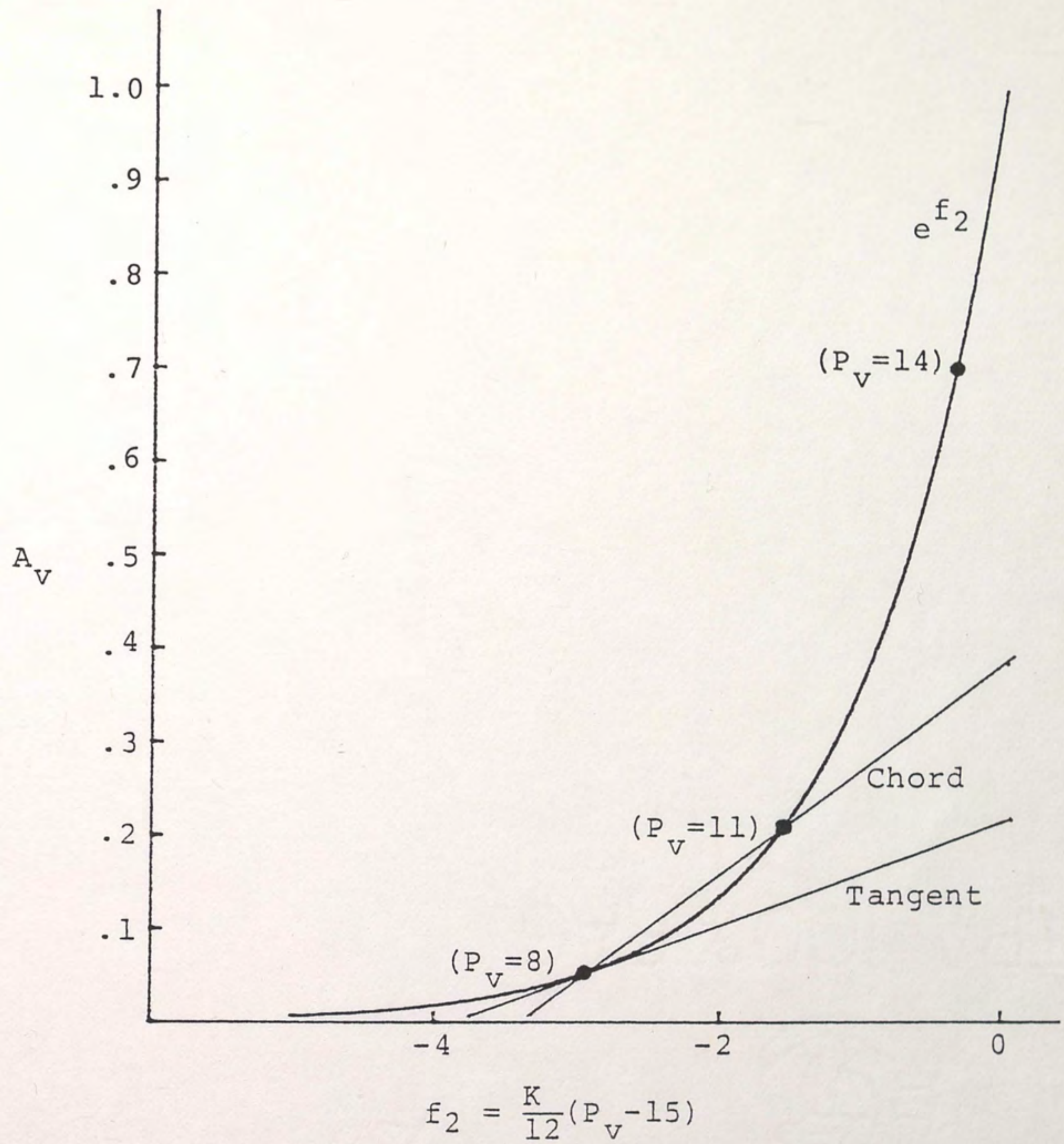


Figure 20. The Exponential Function Describing $A_V$, a Chord Approximation and a Tangent Approximation for the Case When $P_V$ is 8 psi at the Beginning of a Time Step (K=5).

approximation from the exponential function. As with the arbitrary placement of the chord, the two parameters defining the chord line are determined and the solution in NONLINRK would proceed as usual.

Another problem in NONLINRK shown in Chapter 5 is the error that occurs when the control pressure drops below 15 (or rises above 3) in the middle of a large time step. The linear fit routine fixes the inlet valve at full on (or closed when $P_v = 3$) for the entire interval if the system conditions require this at the beginning of the interval. To correct this deficiency it may be worthwhile to recompute the system response breaking this timestep into many small segments to better define the point where the valve transition occurs. Then NONLINRK may be able to resume computation with the original time step and thereby avoid consuming large amounts of computer time.

The final issue deserving more attention is verification of NONLINRK simulations when derivative control is added. A partial verification could be had if the NONLINRK simulations were compared to analytical solutions of the fully linearized system equations for sufficiently small input functions. The case of the sinusoidal variation of setpoint developed in Chapter 4 (see Figure 5) would be an excellent candidate for this analysis. Response amplitude and phase shift predictions would provide a quantitative check of the results.

## APPENDIX I LISTING OF NONLINRK AND SAMPLE OUTPUT

The program NONLINRK is shown on the following pages. This version was formulated for continuous function inputs which are encoded by the user in the subroutines SET(t), SETINT(t), SETDER(t), THETA(t) and PSIN(t). Another version used in this work merely incorporated user prompts for specifications on pulse inputs in place of the encoding process required for continuous function inputs. NONLINRK was run on a Prime 400 minicomputer using single precision variables throughout. The arithmetic operations are accurate to 8 significant figures.

As shown in the program listing, NONLINRK is configured to simulate a sine wave on setpoint pressure. The amplitude is .0546 psi and the period is 5 minutes. Following the model listing is a record of the user prompts and data entry along with model results for the first 2.5 minutes of the simulation. The heading "VAL PSI" refers to the derivative of P, the heading "INTEGRAL" refers to the integral of the error times the reset rate and "A9" and "B9" are the intercept and slope of the linear approximation to the exponential function.

```
OK, ED NONLINRK


OK, ED NONLINRK
EDIT
MODE NUMBER
P 400
.NULL.
        PROGRAM NONLINRK
        REAL K1C, K2D, K4, K5
C       COMPUTE THE RESPONSE OF A FEEDBACK CONTROL APPLIED TO A TANK
C       FOR TIME DEPENDENT INPUT FUNCTIONS DESCRIBING
C       SET POINT PRESSURE, OUTLET % OPENED AND FEED PRESSURE
C
        COMMON A9, B9, P1VBAR, K1C, P2SBAR, T4OLD, T1RIN, C6VHAT, P3ZBAR,
       + K5, C5HAT, T2HEBAR, P9TEST, H2TANK, K4, T7, P7, R1K, K2D, E1RINT, D5ERIV,
       + P4OLD
        CHARACTER YESNO*1
        WRITE (1,150)
        WRITE (1,150)
        WRITE (1,101)
        WRITE (1,102)
        WRITE (1,150)
        WRITE (1,103)
        WRITE (1,150)
        WRITE(1,115)
        WRITE(1,116)
        WRITE(1,150)

        DATA K1C, K2D, T1RIN/3., 0., 1./
        DATA H1BAR, P1VBAR, P3ZBAR, T2HEBAR/30., 9., 100., 50./
        DATA K5, C6VHAT/5., 50./
        DATA H2TANK, RADI/120., 18./

        WRITE (1,104)
        READ *, YESNO
        IF (YESNO .EQ. 'N') GOTO 5
  150   FORMAT (' ')
  101   FORMAT ('  PROGRAM TO COMPUTE RESPONSE OF A PID CONTROLLED TANK',
       +              ' SYSTEM')
  102   FORMAT (20X, 'CONTINUOUS FUNCTION INPUTS')
  103   FORMAT ('  OUTLET VALVE IS SIZED TO YIELD STEADY STATE AT T = 0')
  104   FORMAT ('  CHANGE DATA BASE ? (Y/N) ')
  105   FORMAT ('  INITIAL VALUES OF WATER LEVEL, CONTROL PSI, FEED PSI,
       +            OUTLET % OPENED ')
  106   FORMAT(' ENTER STEPCHANGES TO: TANK SETPOINT, FEED PSI, OUTLET%')
  107   FORMAT(' ENTER GAIN FACTORS: PROPORTIONAL, 1/INTEGRAL, DERIVATIVE')
  108   FORMAT(' ENTER INLET EQUAL % CONSTANT, VALVE SIZE')
  130   FORMAT ('  ENTER TIMESTEP (SEC), NUMBER OF TIMESTEPS TO STOP, ',
       +' NUMBER OF TIMESTEPS PER PRINTED RESULT')
  200   FORMAT (25X, 'MODEL PARAMETERS AND INITIAL CONDITIONS')
  201   FORMAT (7X, 'WATER LEVEL', 4X, '              ', 4X, 'INLET PSI', 4X,
       +'           ', 4X, 'OUT. % OPEN', 4X)
  202   FORMAT(7X, F7.3, 8X, 6X, 8X, F7.2, 7X, 6X, 8X, F5.1)
  203   FORMAT(30X, ' INLET   ', ' EQUAL %  ', ' CONTROL  ', '  OUTLET')
  204   FORMAT(30X, '  SIZE   ', ' CONSTANT  ', '   PSI    ', '   SIZE')
  205   FORMAT(30X, F6.3, 4X, F7.3, 3X, F6.3, 4X, F8.3)
  206   FORMAT(30X, 'PROPORTIONAL ', ' DERIVATIVE ', '        RESET')
  207   FORMAT(30X, '     GAIN     ', '    TIME     ', '        RATE')
  208   FORMAT(32X, E10.3, 3X, E10.3, 5X, E10.3)
  301   FORMAT (4X, 'TIME', '     WATER LEVEL   ', 'TANK PSI', 2X, 'VAL PSI', 7X,
       +'DERIV', 5X, 'INTEGRAL', 6X, 'A9', 10X, 'B9', 9X, 'FLOW IN', 5X, 'FLOW OUT')
  302   FORMAT (2X, F6.2, 3(2X, F10.5), 6(1X, E11.4))
```

```
OK, ED NONLINRK

  303  FORMAT (10(E11.4,1X))
C
C      ENTER NEW DATA BASE
C
       WRITE (1,105)
       READ *,H1BAR,P1VBAR,P3ZBAR,T2HEBAR
       WRITE (1,107)
       READ *,K1C,T1RIN,K2D
       WRITE (1,108)
       READ *,K5,C6VHAT
       GO TO  5
C      END OF NEW DATA ENTRY

    5  WRITE (1,130)
       READ *,D4ELT,I3,I4PRNT
C
C      COMPUTE INITIAL PARAMETERS
C
       T7 = 0
       T4OLD = 0
       E1RINT = 0
       K4 = H2TANK * .03614 * 3.14159 * RADI * RADI / 14.7
       DSERIV = 0
C
C
C
C      ASSUME STEADY STATE AT T = 0 TO SIZE OUTLET VALVE
C
       P2SBAR = H1BAR * 14.7 / (H2TANK - H1BAR)
       C5HAT = C6VHAT * EXP(K5 / 12.  * (P1VBAR-15.))*SQRT(P3ZBAR-P2SBAR)
      1 /(T2HEBAR/100.*SQRT(H1BAR/27.673+P2SBAR))
       WRITE (1,203)
       WRITE (1,204)
       WRITE (1,205) C6VHAT,K5,P1VBAR,C5HAT
       WRITE (1,150)
       WRITE (1,206)
       WRITE (1,207)
       WRITE (1,208) K1C,K2D,T1RIN
       WRITE (1,150)
C
C      PRINT OUTPUT HEADINGS
C
       WRITE(1,150)
       WRITE(1,301)
       IF(A9.GT.1.E-10)GO TO 90
C
C       FIND A9 & B9 AT T = 0
C
       P4OLD = 14.7*H1BAR/(H2TANK-H1BAR)
       P7 = P4OLD
       F2LOWOUT = C5HAT*T2HEBAR/100.*SQRT(.03614*H1BAR+P4OLD)
       F1LOWIN = F2LOWOUT
       CALL LINFIT
       WRITE(1,150)
C
C      - RESULTS AT T=0
C
       WRITE(1,302)T4OLD,H1BAR,P4OLD,P9TEST,D5ERIV,E1RINT,A9,B9,
      @F1LOWIN,F2LOWOUT
   90  CONTINUE
```

```
OK, ED NONLINRK

   95 CONTINUE
C
C     BEGIN SOLUTION LOOP
C
      K6PRN = 0
C     INITIALIZE PRINT INDEX
C
      DO 1000 I = 1, I3
        K6PRN = K6PRN + 1
        R2 = D5ERIV
        P7 = P4OLD + D4ELT / (60.  * 2. ) * R2
        T7 = T4OLD + D4ELT / (60.  * 2. )
        CALL SYSEQ
        R3 = R1K
        P7 = P4OLD+(1. /SQRT(2. )-. 5)*D4ELT*R2/60. +(1. -1. /SQRT(2. ))*
     1 D4ELT*R3/60.
        CALL SYSEQ
        R4 = R1K
        P7 = P4OLD-D4ELT/SQRT(2. )*R3/60. +(1. +1. /SQRT(2. ))*
     1  D4ELT*R4/60.
        T7 = T4OLD + D4ELT/60.
        CALL SYSEQ
        R5 = R1K
        P7 = P4OLD+D4ELT/6. *(R2+(2. -SQRT(2. ))*R3+(2. +SQRT(2. ))
     1  *R4+R5)/60.
C
C     R-K GILL ALGORITHIM COMPLETED FOR THIS TIMESTEP
C
      IF(I. EQ. I3) THEN
       WRITE(1, 150)
       X=FN1(P7)
       Y=FN3(P7)
       WRITE(1, 303)P4OLD, R2, R3, R4, R5, A9, B9, K4, X, Y
       WRITE(1, 150)
      ENDIF

        T4OLD = T4OLD+D4ELT/60.
        E1RINT = E1RINT+(P7+P4OLD)/2. *T1RIN*D4ELT/60.
        EPRNT = T1RIN*SETINT(T4OLD)-E1RINT
        P4OLD = P7
C
C     COMPUTE OUTPUT PARAMETERS, A9 & B9 FOR NEXT STEP
C
        CALL SYSEQ
        D5ERIV = R1K
C
C     USE D5ERIV TO RECOMPUTE A9 & B9
C
  850   CALL LINFIT
C
C     START OF ITERATIVE PROCESS
C     TO IMPROVE A9 & B9 ESTIMATES
C     WHEN DERIVATIVE CONTROL IS EMPLOYED
C     A9 & B9 COMPUTATION COMPLETE IF K2D = 0
C
        IF (K2D . LT.  1. E-10) GOTO 900
        D8TEMP = D5ERIV
        CALL SYSEQ
        D5ERIV = R1K
C
```

OK, ED NONLINRK

```
C      END ITERATION FOR A9, B9 IF CHANGE IN PV IS LESS THAN .2 PSI
C      AS THE DERIVATIVE ESTIMATE IS REFINED ON SUCCESIVE ITERATIONS
C
         IF (ABS(K1C*K2D*(D8TEMP-D5ERIV)).GT. .2) GOTO 850
           CALL LINFIT
  900  CONTINUE
         F1LOWIN=SQRT(PSIN(T4OLD)-P4OLD)*C6VHAT*(A9+B9*K5/12.*(P9TEST-15.
     @ ))
         F2LOWOUT = SQRT(P4OLD+P4OLD*H2TANK*.03614/(14.7+P4OLD))*
     1  C5HAT*THETA(T4OLD)/100.
         D5ERIV = (F1LOWIN-F2LOWOUT)/(K4*FN1(P4OLD))
        CALL LINFIT
         IF (K6PRN .LT. I4PRNT) GOTO 1000
         H9=P4OLD*H2TANK/(14.7+P4OLD)
         WRITE (1,302) T4OLD,H9,P4OLD,P9TEST,D5ERIV,EPRNT,A9,B9,F1LOWIN,
     1  F2LOWOUT
         K6PRN = 0
C      END OF TIMESTEP
 1000  CONTINUE
       END
       FUNCTION SET(T)
       REAL K1C,K2D,K4,K5
       COMMON A9,B9,P1VBAR,K1C,P2SBAR,T4OLD,T1RIN,C6VHAT,P3ZBAR,K5,
     @C5HAT,T2HEBAR,P9TEST,H2TANK,K4,T7,P7,R1K,K2D,E1RINT,D5ERIV,
     @P4OLD
C
C      USER ENCODED TIME DEPENDENT SET POINT PRESSURE
C      FUNCTION- PRESSURE IN PSI, TIME IN MINUTES
C
       SET = 4.9 + .0546*SIN(3.14159/2.5*T)
       RETURN
       END
       FUNCTION SETDER(T)
       REAL K1C,K2D,K4,K5
       COMMON A9,B9,P1VBAR,K1C,P2SBAR,T4OLD,T1RIN,C6VHAT,P3ZBAR,K5,
     @C5HAT,T2HEBAR,P9TEST,H2TANK,K4,T7,P7,R1K,K2D,E1RINT,D5ERIV,
     @P4OLD
C
C      USER ENCODED DERIVATIVE OF SET(T)
C
       SETDER = .0546*3.14159/2.5*COS(3.14159/2.5*T)
       RETURN
       END
       FUNCTION SETINT(T)
       REAL K1C,K2D,K4,K5
       COMMON A9,B9,P1VBAR,K1C,P2SBAR,T4OLD,T1RIN,C6VHAT,P3ZBAR,K5,
     @C5HAT,T2HEBAR,P9TEST,H2TANK,K4,T7,P7,R1K,K2D,E1RINT,D5ERIV,
     @P4OLD
C
C      USER ENCODED INTEGRAL OF SET(T)
C
       SETINT = 4.9*T+.0546*2.5/3.14159*(1.-COS(3.14159/2.5*T))
       RETURN
       END
       FUNCTION THETA(T)
       REAL K1C,K2D,K4,K5
```

```
      COMMON A9,B9,P1VBAR,K1C,P2SBAR,T4OLD,T1RIN,C6VHAT,P3ZBAR,K5,
     @C5HAT,T2HEBAR,P9TEST,H2TANK,K4,T7,P7,R1K,K2D,E1RINT,D5ERIV,
     @P4OLD
C
C     USER ENCODED OUTLET % OPENED FUNCTION OF TIME IN
C     MINUTES
C
      THETA = T2HEBAR
      RETURN
      END
      FUNCTION PSIN(T)
      REAL K1C,K2D,K4,K5
      COMMON A9,B9,P1VBAR,K1C,P2SBAR,T4OLD,T1RIN,C6VHAT,P3ZBAR,K5,
     @C5HAT,T2HEBAR,P9TEST,H2TANK,K4,T7,P7,R1K,K2D,E1RINT,D5ERIV,
     @P4OLD
C
C     USER ENCODED FEED PRESSURE (PSI) FUNCTION OF TIME
C     IN MINUTES
C
      PSIN = P3ZBAR
      RETURN
      END
      FUNCTION FN1(P)

      FN1 = (1-P/(14.7+P))**2
      RETURN
      END

      FUNCTION FN2(P)
      REAL K1C,K2D,K4,K5
      COMMON A9,B9,P1VBAR,K1C,P2SBAR,T4OLD,T1RIN,C6VHAT,P3ZBAR,K5,
     @ C5HAT,T2HEBAR,P9TEST,H2TANK,K4,T7,P7,R1K,K2D,E1RINT,D5ERIV,
     @ P4OLD

      FN2 = SQRT(P*H2TANK*.03614/(14.7+P)+P)
      RETURN
      END

      FUNCTION FN3(P)
      REAL K1C,K2D,K4,K5
      COMMON A9,B9,P1VBAR,K1C,P2SBAR,T4OLD,T1RIN,C6VHAT,P3ZBAR,K5,
     @ C5HAT,T2HEBAR,P9TEST,H2TANK,K4,T7,P7,R1K,K2D,E1RINT,D5ERIV,
     @ P4OLD

      FN3 = SQRT(PSIN(T7)-P)*C6VHAT*B9*K5*K1C*K2D
      FN3 = 1 + FN3/(12.*K4*FN1(P))
      RETURN
      END

      SUBROUTINE SYSEQ
      REAL K1C,K2D,K4,K5
      COMMON A9,B9,P1VBAR,K1C,P2SBAR,T4OLD,T1RIN,C6VHAT,P3ZBAR,K5,
     @ C5HAT,T2HEBAR,P9TEST,H2TANK,K4,T7,P7,R1K,K2D,E1RINT,D5ERIV,
     @ P4OLD

C
C     SYSTEM EQUATION AS SEPARATED FOR dP/dt USING THE
C     LINEAR APPROXIMATION TO THE EXPONENTIAL CONTROL
C     VALVE FUNCTION
C
```

```
OK, ED NONLINRK


 5000 R1K = SET(T7)-P7+T1RIN*SETINT(T7)-E1RINT-T1RIN*(P7+P4OLD)
     1 /2.*(T7-T4OLD)+K2D*SETDER(T7)
       R1K=C6VHAT*SQRT(PSIN(T7)-P7)*(A9+B9*K5*(R1K*K1C+P1VBAR-15.)/12.)
       R1K = R1K-C5HAT*THETA(T7)*FN2(P7)/100.
       R1K = R1K/(FN3(P7)*K4*FN1(P7))
 4000 RETURN
      END

      SUBROUTINE LINFIT
      REAL K1C,K2D,K4,K5
      COMMON A9,B9,P1VBAR,K1C,P2SBAR,T4OLD,T1RIN,C6VHAT,P3ZBAR,K5,
     @ C5HAT,T2HEBAR,P9TEST,H2TANK,K4,T7,P7,R1K,K2D,E1RINT,D5ERIV,
     @ P4OLD
C
C     SUBROUTINE LINFIT COMPUTES THE INTERCEPT (A9) AND
C     SLOPE (B9) OF THE TANGENT TO THE EXPONENTIAL VALVE
C     FUNCTION
C

      A9 = 0
      B9 = 0
      P9TEST = SET(T7)-P7+SETINT(T7)*T1RIN-E1RINT-(P4OLD+P7)/2.*
     1 (T7-T4OLD)*T1RIN+K2D*(SETDER(T7)-D5ERIV)
      P9TEST = P1VBAR+K1C*P9TEST
      IF (P9TEST .GT. 15.) THEN
        P9TEST = 15.
        A9 = 1.0
      ENDIF
      IF(P9TEST .LT. 3.)THEN
        P9TEST = 3.
        A9 = EXP(-K5)
      ENDIF
C
C     RETURN IF VALVE FULL ON/OFF
C
      IF (A9 .GT. 1.E-35) GOTO 100
      B9 = 12./(K5*.002)*(EXP(K5/12.*(P9TEST-14.999))-EXP(K5/12.*
     1 (P9TEST-15.001)))
      A9 = EXP(K5/12.*(P9TEST-14.999))-B9*K5/12.*(P9TEST-14.999)

  100 RETURN
      END
BOTTOM
Q
OK, COMO -END
```

```
PROGRAM TO COMPUTE RESPONSE OF A PID CONTROLLED TANK SYSTEM
                CONTINUOUS FUNCTION INPUTS

 OUTLET VALVE IS SIZED TO YIELD STEADY STATE AT T = 0

 FOR NON-STEADY-STATE INITIAL CONDITIONS, THE SETPOINT AT T=0 IS TAKEN AS THE WATER
 LEVEL AT T=0 PLUS THE STEPCHANGE NEEDED TO ARRIVE AT THE DESIRED SETPOINT (SERVO CASE)

 CHANGE DATA BASE ? (Y/N)
'Y'
 INITIAL VALUES OF WATER LEVEL, CONTROL PSI, FEED PSI,             OUTLET % OPENED
30.,9.,100.,50.
 ENTER GAIN FACTORS: PROPORTIONAL,1/INTEGRAL,DERIVATIVE
3.,1.,1.
 ENTER INLET EQUAL % CONSTANT, VALVE SIZE
5.,50.
 ENTER TIMESTEP (SEC), NUMBER OF TIMESTEPS TO STOP, NUMBER OF TIMESTEPS PER PRINTED RESULT
1.,3600,6
                            INLET    EQUAL %   CONTROL   OUTLET
                             SIZE   CONSTANT     PSI      SIZE
                           50.000      5.000    9.000    32.723

                          PROPORTIONAL   DERIVATIVE      RESET
                              GAIN          TIME          RATE
                            0.300E+01    0.100E+01    0.100E+01
```

| TIME | WATER LEVEL | TANK PSI | VAL PSI | DERIV | INTEGRAL | A9 | B9 | FLOW IN | FLOW OUT |
|---|---|---|---|---|---|---|---|---|---|
| 0.00 | 30.00000 | 4.90000 | 9.20584 | 0.0000E+00 | 0.0000E+00 | 0.3053E+00 | 0.8941E-01 | 0.4002E+02 | 0.4002E+02 |
| 0.10 | 30.00743 | 4.90162 | 9.16895 | 0.1724E-01 | 0.2645E-03 | 0.3020E+00 | 0.8803E-01 | 0.4294E+02 | 0.4003E+02 |
| 0.20 | 30.01555 | 4.90339 | 9.17842 | 0.1821E-01 | 0.1037E-02 | 0.3028E+00 | 0.8837E-01 | 0.4311E+02 | 0.4004E+02 |
| 0.30 | 30.02408 | 4.90525 | 9.18592 | 0.1897E-01 | 0.2292E-02 | 0.3035E+00 | 0.8867E-01 | 0.4325E+02 | 0.4005E+02 |
| 0.40 | 30.03299 | 4.90717 | 9.19128 | 0.1949E-01 | 0.3995E-02 | 0.3039E+00 | 0.8885E-01 | 0.4334E+02 | 0.4005E+02 |
| 0.50 | 30.04192 | 4.90913 | 9.19437 | 0.1978E-01 | 0.6105E-02 | 0.3042E+00 | 0.8898E-01 | 0.4340E+02 | 0.4006E+02 |
| 0.60 | 30.05079 | 4.91111 | 9.19510 | 0.1982E-01 | 0.8572E-02 | 0.3043E+00 | 0.8900E-01 | 0.4341E+02 | 0.4007E+02 |
| 0.70 | 30.06002 | 4.91308 | 9.19342 | 0.1959E-01 | 0.1134E-01 | 0.3042E+00 | 0.8894E-01 | 0.4338E+02 | 0.4008E+02 |
| 0.80 | 30.06889 | 4.91501 | 9.18929 | 0.1911E-01 | 0.1435E-01 | 0.3037E+00 | 0.8876E-01 | 0.4331E+02 | 0.4008E+02 |
| 0.90 | 30.07747 | 4.91689 | 9.18272 | 0.1836E-01 | 0.1754E-01 | 0.3032E+00 | 0.8853E-01 | 0.4319E+02 | 0.4009E+02 |
| 1.00 | 30.08566 | 4.91867 | 9.17377 | 0.1737E-01 | 0.2083E-01 | 0.3023E+00 | 0.8817E-01 | 0.4303E+02 | 0.4010E+02 |
| 1.10 | 30.09333 | 4.92035 | 9.16252 | 0.1614E-01 | 0.2417E-01 | 0.3014E+00 | 0.8780E-01 | 0.4282E+02 | 0.4010E+02 |
| 1.20 | 30.10037 | 4.92189 | 9.14910 | 0.1468E-01 | 0.2747E-01 | 0.3002E+00 | 0.8730E-01 | 0.4258E+02 | 0.4011E+02 |
| 1.30 | 30.10673 | 4.92327 | 9.13366 | 0.1303E-01 | 0.3067E-01 | 0.2989E+00 | 0.8676E-01 | 0.4231E+02 | 0.4012E+02 |
| 1.40 | 30.11229 | 4.92448 | 9.11637 | 0.1120E-01 | 0.3370E-01 | 0.2972E+00 | 0.8610E-01 | 0.4201E+02 | 0.4012E+02 |
| 1.50 | 30.11675 | 4.92550 | 9.09745 | 0.9220E-02 | 0.3648E-01 | 0.2957E+00 | 0.8546E-01 | 0.4168E+02 | 0.4012E+02 |
| 1.60 | 30.12059 | 4.92632 | 9.07716 | 0.7118E-02 | 0.3897E-01 | 0.2939E+00 | 0.8474E-01 | 0.4133E+02 | 0.4013E+02 |
| 1.70 | 30.12343 | 4.92692 | 9.05574 | 0.4925E-02 | 0.4109E-01 | 0.2920E+00 | 0.8397E-01 | 0.4096E+02 | 0.4013E+02 |
| 1.80 | 30.12516 | 4.92729 | 9.03348 | 0.2671E-02 | 0.4279E-01 | 0.2900E+00 | 0.8318E-01 | 0.4058E+02 | 0.4013E+02 |
| 1.90 | 30.12584 | 4.92745 | 9.01068 | 0.3894E-03 | 0.4404E-01 | 0.2881E+00 | 0.8240E-01 | 0.4020E+02 | 0.4013E+02 |
| 2.00 | 30.12549 | 4.92737 | 8.98765 | -0.1888E-02 | 0.4478E-01 | 0.2862E+00 | 0.8163E-01 | 0.3981E+02 | 0.4013E+02 |
| 2.10 | 30.12410 | 4.92706 | 8.96467 | -0.4133E-02 | 0.4498E-01 | 0.2842E+00 | 0.8086E-01 | 0.3943E+02 | 0.4013E+02 |
| 2.20 | 30.12169 | 4.92654 | 8.94208 | -0.6313E-02 | 0.4461E-01 | 0.2823E+00 | 0.8009E-01 | 0.3906E+02 | 0.4013E+02 |
| 2.30 | 30.11831 | 4.92580 | 8.92019 | -0.8399E-02 | 0.4368E-01 | 0.2804E+00 | 0.7936E-01 | 0.3871E+02 | 0.4013E+02 |
| 2.40 | 30.11399 | 4.92486 | 8.89933 | -0.1036E-01 | 0.4216E-01 | 0.2787E+00 | 0.7868E-01 | 0.3838E+02 | 0.4012E+02 |
| 2.50 | 30.10881 | 4.92373 | 8.87976 | -0.1219E-01 | 0.4007E-01 | 0.2770E+00 | 0.7802E-01 | 0.3806E+02 | 0.4012E+02 |

## APPENDIX II TUTSIM MODEL LISTING AND SAMPLE OUTPUT

The model file labelled RAMINOUT.TES and sample output are shown on the following pages. The model includes features to allow the user to input ramp functions in setpoint and/or outlet percentage opening. This version of RAMINOUT.TES is configured for the case of $T_d$ = .5 shown in Figure 19. The output columns show from left to right the time, fluid level in inches, control valve pressure in psi and inlet flow and outlet flow in lbs/minute. The sample output shows the results through 1 minute of simulation for a 1-second time step.

```
MODEL-FILE: RAMINOUT.TES
DATE:          1       1    1980

TUTSIM    ***IBM-PC***
MODEL LISTING

TIMING:    0.0166670      60.0000
PLOTBLOCKS AND RANGES
     Horz:      0      0.0000        100.0000
     53     0.0000        120.0000
     27     0.0000         20.0000
     34     0.0000        500.0000
     45     0.0000        500.0000


MODEL:
  14.7000              1 INT     51                 ;TANK PSI RESULT
   0.0000              2 INT      1                 ;INTEGRAL OF PRESSURE
  14.7000              3 ADL      1
   0.0166670           5 ATT      1     -3          ;TIMESTEP(RESULT=dP/dt ESTIMATE)
                       7 TIM
   0.0000              8 GAI      7                 ;RATE OF SETPT PSI CHANGE
   0.0000              9 LIM      8
   0.0000
  14.7000             10 CON                        ;INITIAL SETPT PSI
   0.0000             16 INT      9     10          ;Ps INTEGRAL
   0.5000000          17 GAI     -2     16          ;RESET RATE x ERROR INTEGRAL
   0.0000             19 CON                        ;dPs/dt AFTER RAMP
   0.0000             20 CON                        ;dPs/dt DURING RAMP
   0.0000             21 REL     19     19     20   ;SELECT CORRECT dPs/DT
                                  7
   0.5000000          23 GAI     -5     21          ;Td x ERROR DERIVATIVE
   3.0000             25 GAI     -1      9     10   ;PROP. GAIN(RESULT=DELTA Pv)
                                 17     23
   9.0000             26 CON                        ;Pv BAR
   3.0000             27 LIM     25     26          ;DELTA Pv AFTER LIMITS CHECK
  15.0000
  15.0000             29 CON                        ;15.
   0.4166670          30 GAI     27    -29          ;EQUAL % CONSTANT/12.
                      31 EXP     30
 100.0000             32 CON                        ;INLET PSI
                      33 SQT     -1     32
                      34 MUL     31     33     54   ;FLOWIN
  80.0000             35 GAI      7                 ;OUTLET RATE %/MIN
   0.0000             36 LIM     35                 ;OUTLET % RAMP MIN&MAX
  80.0000
  10.0000             37 CON                        ;INITIAL OUTLET %
   0.9229400          38 GAI     36     37          ;OUTLET SIZE/100.
 120.0000             39 CON                        ;TANK HEIGHT
   0.0361400          40 CON                        ;WATER DENSITY LB/SQ.INCH
                      41 SUM      1     55          ;(14.7+P)
                      42 MUL      1     39     40   ;DENSxPxHt
                      43 DIV     42     41
                      44 SQT      1     43
                      45 MUL     38     44          ;FLOWOUT
                      46 MUL     41     41          ;(14.7 + P)**2
   1.017E+03          47 CON                        ;TANK AREA
                      48 MUL     39     40     47
                                 55
                      49 DIV     46     48
                      50 SUM     34    -45
                      51 MUL     49     50          ;dP/dt
                      52 MUL      1     39
                      53 DIV     52     41          ;WATER LEVEL RESULT
  50.0000             54 CON                        ;INLET VALVE SIZE
  14.7000             55 CON                        ;ATM PSI
```

PLOTBLOCKS and RANGES
Format: BLOCKNBR,PLOT-MINIMUM,PLOT-MAXIMUM

| | | | |
|---|---|---|---|
| Horz: | 0 | 0.0000 | 100.0000 |
| 53 | 0.0000 | 120.0000 | |
| 27 | 0.0000 | 20.0000 | |
| 34 | 0.0000 | 500.0000 | |
| 45 | 0.0000 | 500.0000 | |

| | | | | |
|---|---|---|---|---|
| 0.0000 | 59.9989 | 9.0000 | 37.9057 | 37.9062 |
| 0.0166670 | 59.9989 | 9.0000 | 37.9057 | 42.9604 |
| 0.0333340 | 59.9966 | 9.1565 | 40.4609 | 48.0122 |
| 0.0500010 | 59.9926 | 9.1867 | 40.9739 | 53.0629 |
| 0.0666680 | 59.9861 | 9.3074 | 43.0871 | 58.1107 |
| 0.0833350 | 59.9786 | 9.3611 | 44.0641 | 63.1565 |
| 0.1000020 | 59.9690 | 9.4679 | 46.0706 | 68.1988 |
| 0.1166690 | 59.9583 | 9.5342 | 47.3624 | 73.2384 |
| 0.1333360 | 59.9447 | 9.6349 | 49.3944 | 78.2740 |
| 0.1500030 | 59.9320 | 9.7081 | 50.9245 | 83.3062 |
| 0.1666700 | 59.9165 | 9.8062 | 53.0513 | 88.3338 |
| 0.1833370 | 59.8999 | 9.8831 | 54.7817 | 93.3572 |
| 0.2000040 | 59.8816 | 9.9802 | 57.0468 | 98.3756 |
| 0.2166710 | 59.8623 | 10.0590 | 58.9545 | 103.3890 |
| 0.2333380 | 59.8415 | 10.1559 | 61.3873 | 108.3980 |
| 0.2500050 | 59.8196 | 10.2352 | 63.4530 | 113.4010 |
| 0.2666720 | 59.7963 | 10.3328 | 66.0899 | 118.4000 |
| 0.2833390 | 59.7720 | 10.4111 | 68.2875 | 123.3890 |
| 0.3000060 | 59.7465 | 10.5102 | 71.1700 | 128.3730 |
| 0.3166730 | 59.7201 | 10.5856 | 73.4480 | 133.3540 |
| 0.3333400 | 59.6923 | 10.6880 | 76.6564 | 138.3260 |
| 0.3500070 | 59.6640 | 10.7576 | 78.9184 | 143.2930 |
| 0.3666740 | 59.6342 | 10.8667 | 82.5948 | 148.2520 |
| 0.3833410 | 59.6041 | 10.9249 | 84.6291 | 153.2050 |
| 0.4000080 | 59.5724 | 11.0484 | 89.1043 | 158.1500 |
| 0.4166750 | 59.5410 | 11.0827 | 90.3980 | 163.0910 |
| 0.4333420 | 59.5073 | 11.2390 | 96.4881 | 168.0210 |
| 0.4500090 | 59.4751 | 11.2199 | 95.7320 | 172.9500 |
| 0.4666760 | 59.4388 | 11.4562 | 105.6460 | 177.8620 |
| 0.4833430 | 59.4073 | 11.3039 | 99.1594 | 182.7820 |
| 0.5000100 | 59.3668 | 11.7525 | 119.5560 | 187.6710 |
| 0.5166770 | 59.3394 | 11.2304 | 96.1878 | 192.5940 |
| 0.5333440 | 59.2893 | 12.2911 | 149.6650 | 197.4420 |
| 0.5500110 | 59.2787 | 10.6111 | 74.3229 | 202.4070 |
| 0.5666780 | 59.2024 | 13.5548 | 253.4600 | 207.1580 |
| 0.5833450 | 59.2629 | 7.5955 | 21.1571 | 212.3500 |
| 0.6000120 | 59.1222 | 15.0000 | 462.9360 | 216.8700 |
| 0.6166790 | 59.3322 | 3.0000 | 3.1173 | 222.5840 |
| 0.6333460 | 59.1270 | 15.0000 | 462.9300 | 226.8580 |
| 0.6500130 | 59.3368 | 3.0000 | 3.1173 | 232.6040 |
| 0.6666800 | 59.1270 | 15.0000 | 462.9300 | 236.8300 |
| 0.6833470 | 59.3323 | 3.0000 | 3.1173 | 242.5910 |
| 0.7000140 | 59.1179 | 15.0000 | 462.9420 | 246.7670 |
| 0.7166810 | 59.3188 | 3.0000 | 3.1174 | 252.5420 |
| 0.7333480 | 59.0998 | 15.0000 | 462.9650 | 256.6650 |
| 0.7500150 | 59.2963 | 3.0000 | 3.1176 | 262.4520 |
| 0.7666820 | 59.0728 | 15.0000 | 462.9990 | 266.5200 |
| 0.7833490 | 59.2649 | 3.0000 | 3.1179 | 272.3170 |
| 0.8000160 | 59.0369 | 15.0000 | 463.0460 | 276.3280 |
| 0.8166830 | 59.2246 | 3.0000 | 3.1182 | 282.1330 |
| 0.8333500 | 58.9921 | 15.0000 | 463.1030 | 286.0860 |
| 0.8500170 | 59.1755 | 3.0000 | 3.1187 | 291.8960 |
| 0.8666840 | 58.9385 | 15.0000 | 463.1710 | 295.7900 |
| 0.8833510 | 59.1176 | 3.0000 | 3.1192 | 301.6030 |
| 0.9000180 | 58.8762 | 15.0000 | 463.2510 | 305.4350 |
| 0.9166850 | 59.0511 | 3.2509 | 3.4635 | 311.2500 |
| 0.9333520 | 58.8053 | 15.0000 | 463.3410 | 315.0200 |
| 0.9500190 | 58.9760 | 3.5843 | 3.9807 | 320.8330 |
| 0.9666860 | 58.7262 | 15.0000 | 463.4410 | 324.5400 |
| 0.9833530 | 58.8925 | 3.9324 | 4.6030 | 330.3510 |
| 1.0000 | 58.6388 | 15.0000 | 463.5520 | 333.9880 |

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